Net reclassification index for ordinal outcome

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Net Reclassification Index for ordinal outcome

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감사의 글

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꼼꼼하고 세심하게 한 단계 한 단계 부족한 부분을 잘 밟아나갈 수 있도록 지도해주신 정인경 교수님, 학문의 무궁무진함을 알게 해주시고 호기심을 갖게 해주신 남정모 교수님, 석사 때부터 한결같이 조언해주시고 관심 가져 주신 송기준 교수님, 바쁘신 와중에도 시간 내어주시고 격려해주신 박소희 교수님, 여러모로 마음 써주시고 배려해주신 허지회 교수님께 진심으로 감사 드립니다. 교수님들이 계셔서 조금이나마 삶의 지혜와 학문의 깊이를 더할 수 있었습니다. 교수님들의 가르침을 바탕으로 겸손하고 꾸준히 학문에 정진하는 제자가 되도록 하겠습니다.

연구부 통계지원실 생활을 한지도 6 년이 다 되어가는데, 그 동안 박사 논문을 잘 쓸 수 있도록 많은 격려와 응원을 아끼지 않았던 부서 선생님들과 원내 선생님들께도 고마움을 전합니다. 많은 배려와 관심으로 박사 과정을 잘 마칠 수 있었습니다. 항상 저를 이끌어 주셨던 의학통계학과 선배님들, 함께

하는 기쁨을 알려준 동기들, 많은 도움을 주었던 의학통계학과 후배님들이 있었기에 많이 배우고 성장할 수 있었고 좋은 추억을 만들 수 있었습니다.

바쁘다는 핑계로 자주 만나지는 못했지만 마음 속 깊이 응원하고 의지할 수 있었던 소중한 친구들이 있었기에 박사 학위 과정 동안 꿋꿋하게 버텨 낼 수 있었습니다. 친구들에게도 감사하다고 전하고 싶습니다.

끝으로 일찍 나가고 늦게 들어오는 딸 때문에 같이 잠 못 이루셨던 어머니, 제가 하는 공부를 묵묵히 뒤에서 응원해주셨던 아버지, 온갖 투정 다 받아주었던 든든한 동생 효근이에게 고맙고 사랑한다고 전하고 싶습니다. 가족이 있어서 편하고 즐겁게 학업에 전념할 수 있었습니다.

지난 7 년의 박사 학위 과정은 많은 것들을 배우고 얻어갈 수 있었던 소중한 시간으로 기억될 것 같습니다. 박사를 마치고 이제 새롭게 첫 발을 내딛는데 매사 책임감 있게 신중하게 행동하고 최선을 다하도록 하겠습니다. 다시 한 번 모든 분들께 감사 드립니다.

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이혜선 올림

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Abstract

In clinical research, identifying new factors that improve predictions of certain disease outcomes is important. Net reclassification improvement (NRI) is a useful measure for assessing the added predictive ability of a new factor. NRI was developed to assess improvements in diagnostic accuracy for various outcomes, including binary, survival, and multiclass outcomes. Ordinal outcomes, such as diagnosis ratings and disease stages, are also important endpoints, for which NRI has not been considered.

In the present study, application of NRI for ordinal outcomes is proposed by extending NRI for binary outcomes and multiclass outcomes with weights that take into account the closeness to the true category when counting reclassification. The standard error of the proposed NRI can be estimated utilizing the variance estimation procedures of the Stuart-Maxwell test and Bhapkar's test statistics.

A simulation study was designed to assess the performance of the proposed method and to compare it with existing methods, such as volume under the receiver operating characteristic surface for ordinal data, reclassification index and NRI for multiclass data, and NRI and the area under the receiver operating characteristic curve for binary data with arbitrary cutoff points. Among the simulation results, the proposed method demonstrated a higher coverage rate for predictive ability than the other methods, especially Delong's method. Also, a simulation setting based on an ordinal structure was more stable than that of a multinomial structure in regards to relative risk. The proposed method was also found to be simple and exhibited a short computing time, while Nakas's method was complex and had a long computing time.

To validate the study results, the noted methods were applied to glaucoma data and nonrelevant cerebral atherosclerosis data. For the glaucoma data, the predictive ability of glaucomatous eyes measured by ΔAUC and NRI using Pencina's method and Delong's method was not improved by adding new factors to existing known factors when considering binary outcomes according to arbitrary cutoff points. Meanwhile, NRI using the newly proposed method led to improved predictive ability with the addition of new factors to existing known factors. For nonrelevant cerebral atherosclerosis data, applying NRI by the newly proposed method revealed improvement in reclassification with the addition of the presence of nonrelevant cerebral atherosclerosis or burden of nonrelevant cerebral atherosclerosis when the original categories of seven was kept; however, the significance of NRI was not shown in other categories divided arbitrarily. In other words, the predictive ability increased when keeping the original categories of the ordinal outcomes.

The newly proposed method for ordinal outcomes described in the present study is a useful discriminant measure with a short computing time and it is simple to interpret. Therefore, the method is more readily applicable to real data than other existing methods.

Keywords: new factor, ordinal outcome, NRI, VUS

I. Introduction

1.1 Net reclassification index

Identifying new factors that improve the predictive ability of existing known factors is important in clinical research. For example, tumor size, cancer stage, and lymph node involvement have been shown to be factors predictive of cancer recurrence. Meanwhile, discovery of new factors that can be used together with these known factors holds the potential to improve the accuracy thereof.

Area under the receiver operating characteristic curve (AUC), volume under the receiver operating characteristic surface (VUS), net reclassification improvement (NRI), and integrated discrimination improvement (IDI) are known to assessment measures of whether a new factor will predict outcomes well. Net reclassification improvement is an especially useful statistical tool for evaluating predictive accuracy. NRI is also intuitive and easy to interpret. (Pencina et al, 2008). The measure can be used to compare a model including all factors (full model) and another including all factors except for the one new factor (reduced model). After calculating the predicted probability of the two models, one can make a contingency table according to disease status; contingency tables with dimensions of $2x2$, $3x3$, or $4x4$ are normally used. Rows represent categories based on the predictive probability of the reduced model, and columns represent categories based on the predictive probability of the full model.

Taking binary outcomes to indicate disease status, if a category classified by the predictive probability of the full model is higher than a category classified by the predictive probability of the reduced model in the disease group, then one would conclude an improvement in sensitivity. On the other hand, in the non-disease group, if a category classified by the predictive probability of the full model is lower than a category classified by the predictive probability of then reduced model, it would indicate an improvement in specificity.

NRI has been developed to assess improvements in diagnostic accuracy for various outcomes, such as binary (e.g., disease status) (Pencina et al., 2008), survival (e.g., overall survival, disease free survival) (Uno et al., 2012; Zheng et al., 2013), and multiclass outcomes (e.g., disease type) (Li et al., 2013). Nevertheless, ordinal outcomes, such as diagnosis ratings and disease stages, are also important endpoints, for which NRI has not yet been considered.

1.2 Ordinal category data

The usefulness of the ordinal category outcomes is well known in medical and public health fields. Examples of ordinal outcomes include diagnostic ratings based on a mammogram to detect breast cancer (definitely normal, probably normal, equivocal, probably abnormal, definitely abnormal), quality of life (never, rarely, occasionally, often), pain or symptom severity (none, mild, moderate, severe), illness after a period of treatment (much worse, a bit worse, the same, a bit better, much better), and stages of disease (I, II, III). When innately continuous variables are summarized by collapsing all values into a set of categories, ordinal scale outcomes are created: for example, body mass index measured as <18.5, 18.5-24.9, 25-29.9, and ≥ 30 for underweight, normal weight, overweight, and obesity and systolic blood pressure measured as <120, 120-139, 140-159, and $\geq$160 for normal, prehypertension, stage I hypertension, and stage 2 hypertension, respectively.

Data analyses for ordinal outcomes utilize multinomial logistic regression or ordinal logistic regression. Ordinal logistic analyses, in particular, have many advantages. Use of ordinal analyses facilities easier interpretation and is more parsimonious than multinomial logistic models. Ordinal analyses also potentially offer better power than an analysis ignoring of the ordinarily (Agresti, 2010).

1.3 Objectives and Outline

In the present study, application of NRI for ordinal outcomes is proposed by extending NRI for binary outcomes and multiclass outcomes with weights that take into account the closeness to the true category when counting reclassification. The standard error of the proposed NRI can be estimated utilizing the variance estimation procedures of Stuart-Maxwell test and Bhapkar's test statistics (Stuart, 1955; Maxwell, 1970; Sun et al., 2008).

The theoretical framework of NRI and AUC are described in Section II. The description covers the existing methods for computing AUC, VUS, and NRI, as well as, the notation, estimate, and computation of the existing standard error. In Section III, the estimation method for ordinal outcomes is provided. Here, the notations that are used for estimating NRI and the standard error (SE) of NRI for three-category outcomes are also described, followed by an explanation of the estimation and inference. In Section IV, all of the results from the simulations are compared to assess the performance of the proposed method. Next, in Section V, the newly proposed NRI method is applied to glaucoma data and nonrelevant cerebral atherosclerosis data. Finally, the present study concludes with a discussion in Section VI.

II. Literature Review

2.1 Area under the ROC curve and volume under the surface

2.1.1 Binary outcomes

To assess the added predictive ability of a new factor, two models are considered. Let M_1 mean a model with existing known factors and M_2 mean a model with existing known factors and a new factor. The two models are obtained by binary logistic regression (Agresti, 2007). Let Y be a binary outcome that takes on the values 1 and 2, which mean non-event and event, respectively.

To predict the event, M_1 uses p risk factors and M_2 uses $p+1$ risk factors, and we denote as $\boldsymbol{x}_{\boldsymbol{M_1}} = (x_1, \cdots, x_p)'$, $\boldsymbol{x}_{\boldsymbol{M_2}} = (x_1, \cdots, x_p, x_{p+1})'$. We produce linear coefficients estimates $\boldsymbol{\beta}_{\boldsymbol{M_1}} = (\beta_{1,1}, \cdots, \beta_{p,1})'$ for M_1 and $\boldsymbol{\beta}_{\boldsymbol{M_2}} = (\beta_{1,2}, \cdots, \beta_{p,2}, \beta_{p+1,2})'$ for M_2.

The two models are

$$M_1\text{: } \mathrm{logit}\big(P(Y=2)\big) = \ln\left(\frac{P(Y=2)}{1-P(Y=2)}\right)$$

$$= \alpha_1 + \beta_{1,1}x_1 + \cdots + \beta_{p,1}x_p = \alpha_1 + \boldsymbol{\beta}'_{\boldsymbol{M_1}}\boldsymbol{x}_{\boldsymbol{M_1}},$$

$$M_2\text{: } \mathrm{logit}\big(P(Y=2)\big) = \ln\left(\frac{P(Y=2)}{1-P(Y=2)}\right)$$

$$= \alpha_2 + \beta_{1,2}x_1 + \cdots + \beta_{p,2}x_n + \beta_{p+1,2}x_{p+1} = \alpha_2 + \boldsymbol{\beta}'_{\boldsymbol{M_2}}\boldsymbol{x}_{\boldsymbol{M_2}},$$

and the predicted probabilities for an event are gained from M_1 and M_2 as follows:

$$M_1\colon P_{M_1} = P(Y=2) = \frac{\exp\left(\alpha_1 + \boldsymbol{\beta}'_{\boldsymbol{M_1}}\boldsymbol{x}_{\boldsymbol{M_1}}\right)}{1+\exp\left(\alpha_1 + \boldsymbol{\beta}'_{\boldsymbol{M_1}}\boldsymbol{x}_{\boldsymbol{M_1}}\right)},$$

$$M_2\colon P_{M_2} = P(Y=2) = \frac{\exp\left(\alpha_2 + \boldsymbol{\beta}'_{\boldsymbol{M_2}}\boldsymbol{x}_{\boldsymbol{M_2}}\right)}{1+\exp\left(\alpha_2 + \boldsymbol{\beta}'_{\boldsymbol{M_2}}\boldsymbol{x}_{\boldsymbol{M_2}}\right)}.$$

DeLong et al. (1988) proposed comparing the discriminant ability of two models for binary outcomes using predicted probability obtained from M_1 and M_2. AUCs based on M_1 and M_2 are defined as

AUC based on M_1: $\theta_{M_1} = P\left[P_{M_{1,}1} < P_{M_{1,}2}\right]$,

AUC based on M_2: $\theta_{M_2} = P[P_{M_{2,}1} < P_{M_{2,}2}]$.

Two AUCs, θ_{M_1} and θ_{M_2}, are equal to the probability that the two measurements, one from each outcome category, will be in the correct order.

Let $P_{M_1,1,i}$ and $P_{M_1,2,j}$ denote the predicted probability of M_1 in the i^{th} non-event group subject $(i = 1, \cdots, m)$ and the predicted probability of M_1 in the j^{th} event group subject $(j = 1, \cdots, n)$. Let $P_{M_2,1,i}$ and $P_{M_2,2,j}$ denote the predicted probability of M_2 in the i^{th} non-event group subject $(i = 1, \cdots, m)$ and the predicted probability of M_2 in the j^{th} event group subject $(j = 1, \cdots, n)$. m and n are the sizes of the event group and non-event group respectively. Two AUCs are estimated for M_1 and M_2 as follows:

$\widehat{AUC}$ based on M_1: $\hat{\theta}_{M_1,b} = \frac{1}{mn}\sum_{i=1}^{m}\sum_{j=1}^{n} I(P_{M_1,1,i}, P_{M_1,2,j})$

$$I(P_{M_1,1,i}, P_{M_1,2,j}) = \begin{cases} 1, if\ P_{M_1,1,i} < P_{M_1,2,j} \\ \frac{1}{2}, if\ P_{M_1,1,i} = P_{M_1,2,j}, \\ 0, if\ P_{M_1,1,i} > P_{M_1,2,j} \end{cases}$$

$\widehat{AUC}$ based on M_2: $\hat{\theta}_{M_2,b} = \frac{1}{mn}\sum_{i=1}^{m}\sum_{j=1}^{n} I(P_{M_2,1,i}, P_{M_2,2,j})$

$$I(P_{M_2,1,i}, P_{M_2,2,j}) = \begin{cases} 1, if\ P_{M_2,1,i} < P_{M_2,2,j} \\ \frac{1}{2}, if\ P_{M_2,1,i} = P_{M_2,2,j}, \\ 0, if\ P_{M_2,1,i} > P_{M_2,2,j} \end{cases}$$

where, $I(P_{M_1,1,i}, P_{M_1,2,j})$ and $I(P_{M_2,1,i}, P_{M_2,2,j})$ are indicator function adjusted for ties. The indicator function gives a score by comparing the predicted probability in randomly selected subjects from the event group with the predicted probability from the non-event group. If the predicted probability in randomly selected subjects from the event group is larger than the one from the non-event group, the indicator function gives a higher score. Expected values of $\hat{\theta}_{M_1,b}$ and $\hat{\theta}_{M_2,b}$ are

$$E(\hat{\theta}_{M_1,b}) = P(P_{M_1,1,i} < P_{M_1,2,j}) + \frac{1}{2}P(P_{M_1,1,i} = P_{M_1,2,j}),$$

$$E(\hat{\theta}_{M_2,b}) = P(P_{M_2,1,i} < P_{M_2,2,j}) + \frac{1}{2}P(P_{M_2,1,i} = P_{M_2,2,j}).$$

$Var(\hat{\theta}_{M_1,b}), Var(\hat{\theta}_{M_2,b})$, and r are used for variance estimation for $(\hat{\theta}_{M_2,b} - \hat{\theta}_{M_1,b})$. Variance of $\widehat{AUC}$ based on M_1 is

$$Var(\,\hat{\theta}_{M_1,b}) = \sigma^2_{M_1,b} = \frac{1}{mn}[\theta_{M_1,b}(1-\theta_{M_1,b}) + (n-1)(q_{M_1,1} - {\theta_{M_1,b}}^2)$$
$$+(m-1)(q_{M_1,2} - {\theta_{M_1,b}}^2)],$$

$$q_{M_1,1} = P\left[I(P_{M_1,1,i}, P_{M_1,2,j}) = I\left(P_{M_1,1,i}, P_{M_1,2,j'}\right) = 1\right], j \neq j',$$
$$q_{M_1,2} = P\left[I(P_{M_1,1,i}, P_{M_1,2,j}) = I\left(P_{M_1,1,i'}, P_{M_1,2,j}\right) = 1\right], i \neq i'.$$

where, $\hat{q}_{M_1,1}$ is the probability of correctly scoring for two subjects $P_{M_1,1,i}$, $P_{M_1,2,j}$ and correctly scoring $P_{M_1,1,i}$ and different $Y=2$ subject $P_{M_1,2,j'}$ in M_1. $\hat{q}_{M_1,2}$ has interpretations similar to $\hat{q}_{M_1,1}$. Estimation for $\hat{q}_{M_1,1}$ and $\hat{q}_{M_1,2}$ are obtained as follows:

$$\hat{q}_{M_1,1} = \frac{1}{mn(n-1)} \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{\substack{j=1 \\ j \neq j'}}^{n} I(P_{M_1,1,i}, P_{M_1,2,j}) I(P_{M_1,1,i}, P_{M_1,2,j'}),$$
$$\hat{q}_{M_1,2} = \frac{1}{mn(m-1)} \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{\substack{i=1 \\ i \neq i'}}^{m} I\,(P_{M_1,1,i}, P_{M_1,2,j}) I(\,P_{M_1,1,i'}, P_{M_1,2,j}).$$

Variance of $\widehat{\mathrm{AUC}}$ based on M_2 is

$$Var(\,\hat{\theta}_{M_2,b}) = \sigma^2_{M_2,b} = \frac{1}{mn}[\theta_{M_2,b}(1-\theta_{M_2,b}) + (n-1)(q_{M_2,1} - {\theta_{M_2,b}}^2)$$
$$+(m-1)(q_{M_2,2} - {\theta_{M_2,b}}^2)],$$

$$q_{M_1,1} = P\left[I(P_{M_2,1,i}, P_{M_2,2,j}) = I\left(P_{M_2,1,i}, P_{M_2,2,j'}\right) = 1\right], j \neq j',$$

$$q_{M_2,2} = P\left[I(P_{M_2,1,i}, P_{M_2,2,j}) = I\left(P_{M_2,1,i'}, P_{M_2,2,j}\right) = 1\right], i \neq i',$$

where, $q_{M_2,1}$ is the probability of the correctly scoring of two subjects $P_{M_2,1,i}$, $P_{M_2,2,j}$ and correctly scoring $P_{M_2,1,i}$ and different $Y = 2$ subject $P_{M_2,2,j'}$ in M_2. $q_{M_2,2}$ has interpretations similar to $q_{M_2,1}$. Estimation for $q_{M_2,1}$ and $q_{M_2,2}$ are obtained as follows:

$$\hat{q}_{M_2,1} = \frac{1}{mn(n-1)} \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{\substack{j=1 \\ j \neq j'}}^{n} I(P_{M_2,1,i}, P_{M_2,2,j}) I(P_{M_2,1,i}, P_{M_2,2,j'}),$$

$$\hat{q}_{M_2,2} = \frac{1}{mn(m-1)} \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{\substack{i=1 \\ i \neq i'}}^{m} I\left(P_{M_2,1,i}, P_{M_2,2,j}\right) I\left(P_{M_2,1,i'}, P_{M_2,2,j}\right).$$

r is the ratio of $Cov(\hat{\theta}_{M_1,b}, \hat{\theta}_{M_2,b})$, and $\sigma_{M_1,b}\sigma_{M_2,b}$, it is calculated as follows:

$$r = \frac{Cov(\hat{\theta}_{M_1,b}, \hat{\theta}_{M_2,b})}{\sigma_{M_1,b}\sigma_{M_2,b}},$$

where,

$$Cov(\hat{\theta}_{M_1,b}, \hat{\theta}_{M_2,b}) = \frac{1}{mn}[q_{12} - \theta_{M_1,b}\theta_{M_2,b} + (n-1)(q_1 - \theta_{M_1,b}\theta_{M_2,b})$$
$$+(m-1)(q_2 - \theta_{M_1,b}\theta_{M_2,b})],$$

$$q_{12} = P\left[I(P_{M_1,1,i}, P_{M_1,2,j}) = I(P_{M_2,1,i}, P_{M_2,2,j}) = 1\right],$$

$$q_1 = P\left[I(P_{M_{1},1,i}, P_{M_{1},2,j}) = I\left(P_{M_{2},1,i}, P_{M_{2},2,j'}\right) = 1\right], j \neq j',$$

$$q_2 = P\left[I(P_{M_{1},1,i}, P_{M_{1},2,j}) = I\left(P_{M_{2},1,i'}, P_{M_{2},2,j}\right) = 1\right], i \neq i'.$$

q_{12} is the probability of the correctly scoring of two subjects $P_{M_{1},1,i}, P_{M_{1},2,j}$ in M_1 and correctly scoring $P_{M_{2},1,i}, P_{M_{2},2,j}$ in M_2. q_1 is the probability of correctly scoring for two subjects $P_{M_{1},1,i}, P_{M_{1},2,j}$ in M_1 and correctly scoring $P_{M_{2},1,i}$ and different $Y = 2$ subject $P_{M_{2},2,j'}$ in M_2. q_2 has interpretations similar to q_1. Estimation for q_{12}, q_1 and q_2 are obtained as follows:

$$\hat{q}_{12} = \frac{1}{mn}\sum_{i=1}^{m}\sum_{j=1}^{n} I(P_{M_{1},1,i}, P_{M_{1},2,j})\, I(P_{M_{2},1,i}, P_{M_{2},2,j}),$$

$$\hat{q}_1 = \frac{1}{mn(n-1)}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{\substack{j=1 \\ j\neq j'}}^{n} I(P_{M_{1},1,i}, P_{M_{1},2,j})\, I\,(P_{M_{2},1,i}, P_{M_{2},2,j'}),$$

$$\hat{q}_2 = \frac{1}{mn(m-1)}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{\substack{i=1 \\ i\neq i\prime}}^{m} I(P_{M_{1},1,i}, P_{M_{1},2,j}) I(P_{M_{2},1,i'}, P_{M_{2},2,j}).$$

$Z_{\Delta AUC}$ is the test statistic for testing the null hypothesis which is $\Delta AUC = 0$, and it is calculated as shown below:

$$Z_{\Delta AUC} = \frac{\hat{\theta}_{M_2,b} - \hat{\theta}_{M_1,b}}{\sqrt{\hat{\sigma}^2_{M_1,b} + \hat{\sigma}^2_{M_2,b} - 2\hat{r}\hat{\sigma}^2_{M_1,b}\hat{\sigma}^2_{M_2,b}}},$$

and compared to a standard normal distribution.

2.1.2 Ordinal outcomes

To assess the added predictive ability of a new factor for ordinal outcomes, the notations M_1 and M_2 are used with the same meaning as the one used in 2.1.1 binary outcomes. Two models are obtained via multinomial logistic regression or ordinal logistic regression (Agresti, 2007). Let Y refer to an ordinal outcome, $Y = \{1,2,\cdots,c\}$, with symptom severity increasing, toward c. To predict the severity of the symptom, M_1 uses p risk factors and M_2 uses $p+1$ risk factors, and we denote as $\boldsymbol{x}_{\boldsymbol{M_1}} = (x_1,\cdots,x_p)'$, $\boldsymbol{x}_{\boldsymbol{M_2}} = (x_1,\cdots,x_p,x_{p+1})'$. We then produce linear coefficient estimates $\boldsymbol{\beta}_{\boldsymbol{c},\boldsymbol{M_1}} = (\beta_{1,c.1},\cdots,\beta_{p,c,1})'$ for M_1 and $\boldsymbol{\beta}_{\boldsymbol{c},\boldsymbol{M_2}} = (\beta_{1,c,2},\cdots,\beta_{p,c,2},\beta_{p+1,c,2})'$ for M_2 in multinomial logistic regression, and $\boldsymbol{\beta}_{\boldsymbol{M_1}} = (\beta_{1,1},\cdots,\beta_{p,1})'$ for M_1 and $\boldsymbol{\beta}_{\boldsymbol{M_2}} = (\beta_{1,2},\cdots,\beta_{p,2},\beta_{p+1,2})'$ for M_2 in ordinal logistic regression.

When using multinomial logistic regression, the two models are

$$M_1\text{: logit}[P(Y=c)] = \ln\left[\frac{P(Y=c)}{P(Y=C)}\right] = \ln\left[\frac{\pi_c}{\pi_C}\right], c = 1,\cdots,C-1$$

$$= \alpha_{c,1} + \beta_{1,c,1}x_1 + \cdots + \beta_{p,c,1}x_p,$$

$$M_2\text{: logit}[P(Y=c)] = \ln\left[\frac{P(Y=c)}{P(Y=C)}\right] = \ln\left[\frac{\pi_c}{\pi_C}\right], c = 1,\cdots,C-1$$

$$= \alpha_{c,2} + \beta_{1,c,2}x_1 + \cdots + \beta_{p,c,2}x_p + \beta_{p+1,c,2}x_{p+1},$$

where, π_c is the probability of the c^{th} category and C is a reference category. Each subject in the dataset has three predicted probabilities of the event gained from M_1 and M_2 as follows:

$$M_1: P_{M_1}(Y=c) = \frac{\exp\left(\alpha_{c,1} + \boldsymbol{\beta}'_{\boldsymbol{c},\boldsymbol{M_1}} \boldsymbol{x}_{\boldsymbol{M_1}}\right)}{1 + \sum_{c=1}^{c-1} \exp\left(\alpha_{c,1} + \boldsymbol{\beta}'_{\boldsymbol{c},\boldsymbol{M_1}} \boldsymbol{x}_{\boldsymbol{M_1}}\right)},$$

$$M_2: P_{M_2}(Y=c) = \frac{\exp\left(\alpha_{c,2} + \boldsymbol{\beta}'_{\boldsymbol{c},\boldsymbol{M_2}} \boldsymbol{x}_{\boldsymbol{M_2}}\right)}{1 + \sum_{c=1}^{c-1} \exp\left(\alpha_{c,2} + \boldsymbol{\beta}'_{\boldsymbol{c},\boldsymbol{M_2}} \boldsymbol{x}_{\boldsymbol{M_2}}\right)}.$$

For example, predicted probabilities for three categories are

M_1:

$$p_1(M_1) = P_{M_1}(Y=1) = \frac{\exp\left(\alpha_{1,1} + \boldsymbol{\beta}'_{\boldsymbol{1},\boldsymbol{M_1}} \boldsymbol{x}_{\boldsymbol{M_1}}\right)}{1 + \exp\left(\alpha_{1,1} + \boldsymbol{\beta}'_{\boldsymbol{1},\boldsymbol{M_1}} \boldsymbol{x}_{\boldsymbol{M_1}}\right) + \exp\left(\alpha_{2,1} + \boldsymbol{\beta}'_{\boldsymbol{2},\boldsymbol{M_1}} \boldsymbol{x}_{\boldsymbol{M_1}}\right)},$$

$$p_2(M_1) = P_{M_1}(Y=2) = \frac{\exp\left(\alpha_{2,1} + \boldsymbol{\beta}'_{\boldsymbol{2},\boldsymbol{M_1}} \boldsymbol{x}_{\boldsymbol{M_1}}\right)}{1 + \exp\left(\alpha_{1,1} + \boldsymbol{\beta}'_{\boldsymbol{1},\boldsymbol{M_1}} \boldsymbol{x}_{\boldsymbol{M_1}}\right) + \exp\left(\alpha_{2,1} + \boldsymbol{\beta}'_{\boldsymbol{2},\boldsymbol{M_1}} \boldsymbol{x}_{\boldsymbol{M_1}}\right)},$$

$$p_3(M_1) = P_{M_1}(Y=3) = \frac{1}{1 + \exp\left(\alpha_{1,1} + \boldsymbol{\beta}'_{\boldsymbol{1},\boldsymbol{M_1}} \boldsymbol{x}_{\boldsymbol{M_1}}\right) + \exp\left(\alpha_{2,1} + \boldsymbol{\beta}'_{\boldsymbol{2},\boldsymbol{M_1}} \boldsymbol{x}_{\boldsymbol{M_1}}\right)},$$

M_2:

$$p_1(M_2) = P_{M_2}(Y=1) = \frac{\exp\left(\alpha_{1,2} + \boldsymbol{\beta}'_{\boldsymbol{1},\boldsymbol{M_2}} \boldsymbol{x}_{\boldsymbol{M_2}}\right)}{1 + \exp\left(\alpha_{1,2} + \boldsymbol{\beta}'_{\boldsymbol{1},\boldsymbol{M_2}} \boldsymbol{x}_{\boldsymbol{M_2}}\right) + \exp\left(\alpha_{2,2} + \boldsymbol{\beta}'_{\boldsymbol{2},\boldsymbol{M_2}} \boldsymbol{x}_{\boldsymbol{M_2}}\right)},$$

$$p_2(M_2) = P_{M_2}(Y=2) = \frac{\exp\left(\alpha_{2,2} + \boldsymbol{\beta}'_{\boldsymbol{2},\boldsymbol{M_2}} \boldsymbol{x}_{\boldsymbol{M_2}}\right)}{1 + \exp\left(\alpha_{1,2} + \boldsymbol{\beta}'_{\boldsymbol{1},\boldsymbol{M_2}} \boldsymbol{x}_{\boldsymbol{M_2}}\right) + \exp\left(\alpha_{2,2} + \boldsymbol{\beta}'_{\boldsymbol{2},\boldsymbol{M_2}} \boldsymbol{x}_{\boldsymbol{M_2}}\right)},$$

$$p_3(M_2) = P_{M_2}(Y=3) = \frac{1}{1 + \exp\left(\alpha_{1,2} + \boldsymbol{\beta}'_{\boldsymbol{1},\boldsymbol{M_2}} \boldsymbol{x}_{\boldsymbol{M_2}}\right) + \exp\left(\alpha_{2,2} + \boldsymbol{\beta}'_{\boldsymbol{2},\boldsymbol{M_2}} \boldsymbol{x}_{\boldsymbol{M_2}}\right)}.$$

When using ordinal logistic regression, the two models are

$$M_1\text{: logit}[P(Y \leq c)] = \ln\left[\frac{P(Y \leq c)}{1 - P(Y \leq c)}\right] = \ln\left[\frac{\pi_{1,1} + \cdots + \pi_{c,1}}{\pi_{c+1,1} + \cdots + \pi_{C,1}}\right]$$

$$= \alpha_{c,1} + \beta_{1,1}x_1 + \cdots + \beta_{p,1}x_p = \alpha_1 + \boldsymbol{\beta}'_{\boldsymbol{M_1}}\boldsymbol{x}_{\boldsymbol{M_1}},$$

$$M_2\text{: logit}[P(Y \leq c)] = \ln\left[\frac{P(Y \leq c)}{1 - P(Y \leq c)}\right] = \ln\left[\frac{\pi_{1,2} + \cdots + \pi_{c,2}}{\pi_{c+1,2} + \cdots + \pi_{C,2}}\right]$$

$$= \alpha_{c,2} + \beta_{1,2}x_1 + \cdots + \beta_{p,2}x_p \ + \beta_{p+1,2}x_{p+1} = \alpha_2 + \boldsymbol{\beta}'_{\boldsymbol{M_2}}\boldsymbol{x}_{\boldsymbol{M_2}},$$

where, π_c is the probability of the c^{th} category. Predicted probabilities of each category for M_1 and M_2 are obtained from ordinal logistic regression model as follows:

$$M_1\text{:}\, P_{M_1}(Y = c) = \frac{\exp(\alpha_{c,1} + \boldsymbol{\beta}'_{\boldsymbol{M_1}}\boldsymbol{x}_{\boldsymbol{M_1}})}{1 + \exp(\alpha_{c,1} + \boldsymbol{\beta}'_{\boldsymbol{M_1}}\boldsymbol{x}_{\boldsymbol{M_1}})} - P_{M_1}(Y \leq c - 1),$$

$$M_2\text{:}\, P_{M_2}(Y = c) = \frac{\exp(\alpha_{c,2} + \boldsymbol{\beta}'_{\boldsymbol{M_2}}\boldsymbol{x}_{\boldsymbol{M_2}})}{1 + \exp(\alpha_{c,2} + \boldsymbol{\beta}'_{\boldsymbol{M_2}}\boldsymbol{x}_{\boldsymbol{M_2}})} - P_{M_2}(Y \leq c - 1).$$

For example, the predicted probability for three categories is

M_1:

$$p_1(M_1) = P_{M_1}(Y = 1) = \frac{\exp(\alpha_{1,1} + \boldsymbol{\beta}'_{\boldsymbol{M_1}}\boldsymbol{x}_{\boldsymbol{M_1}})}{1 + \exp(\alpha_{1,1} + \boldsymbol{\beta}'_{\boldsymbol{M_1}}\boldsymbol{x}_{\boldsymbol{M_1}})},$$

$$p_2(M_1) = P_{M_1}(Y = 2) = \frac{\exp(\alpha_{1,2} + \boldsymbol{\beta}'_{\boldsymbol{M_1}}\boldsymbol{x}_{\boldsymbol{M_1}})}{1 + \exp(\alpha_{1,2} + \boldsymbol{\beta}'_{\boldsymbol{M_1}}\boldsymbol{x}_{\boldsymbol{M_1}})} - P_{M_1}(Y \leq 1),$$

$$p_3(M_1) = P_{M_1}(Y = 3) = 1 - P_{M_1}(Y \leq 2),$$

M_2:

$$p_1(M_2) = P_{M_2}(Y=1) = \frac{\exp(\alpha_1 + \boldsymbol{\beta}'_{\boldsymbol{M_2}}\boldsymbol{x}_{\boldsymbol{M_2}})}{1+\exp(\alpha_1 + \boldsymbol{\beta}'_{\boldsymbol{M_2}}\boldsymbol{x}_{\boldsymbol{M_2}})},$$

$$p_2(M_2) = P_{M_2}(Y=2) = \frac{\exp(\alpha_2 + \boldsymbol{\beta}'_{\boldsymbol{M_2}}\boldsymbol{x}_{\boldsymbol{M_2}})}{1+\exp(\alpha_2 + \boldsymbol{\beta}'_{\boldsymbol{M_2}}\boldsymbol{x}_{\boldsymbol{M_2}})} - P_{M_2}(Y \le 1),$$

$$p_3(M_2) = P_{M_2}(Y=3) = 1 - P_{M_2}(Y \le 2).$$

Dreiseitl et al. (2000) and Nakas et al. (2004, 2014) proposed comparing discriminant ability for ordinal outcomes using predicted probability obtained from M_1 and M_2.

To do so, let $P_{M_1,1,i}$ be a predicted category by M_1 when subjects are from Y=1 $(i = 1, \cdots, m)$, $P_{M_1,2,j}$ be a predicted category by M_1 when subjects are from Y=2 $(j = 1, \cdots, n)$ and $P_{M_1,3,k}$ be a predicted category by M_1 when subjects are from Y=3 $(k = 1, \cdots, l)$. Let $P_{M_2,1,i}$ be a predicted category by M_2 when subjects are from Y=1 $(i = 1, \cdots, m)$, $P_{M_2,2,j}$ be a predicted category by M_2 when subjects are from Y=2 $(j = 1, \cdots, n)$, and $P_{M_2 3,k}$ be a predicted category by M_2 when subjects are from Y=3 $(k = 1, \cdots, l)$. The numbers of $Y = 1,2,3$ are m, n and l respectively.

Two VUSs based on M_1 and M_2 are defined

VUS based on M_1: $\theta_{M_1} = P[P_{M_1,1} < P_{M_1,2} \cap P_{M_1,2} < P_{M_1,3}]$,

VUS based on M_2: $\theta_{M_2} = P[P_{M_2,1} < P_{M_2,2} \cap P_{M_2,2} < P_{M_2,3}]$.

Two VUSs, θ_{M_1} and θ_{M_2}, are equal to the probability that three measurements, one from each outcome category, will be in the correct order.

Two VUSs of M_1 and M_2 are estimated as

$$\widehat{\text{VUS}} \text{ based on } M_1: \hat{\theta}_{M_1,o} = \frac{1}{mnl}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l} I(P_{M_1,1,i}, P_{M_1,2,j}, P_{M_1,3,k}),$$

$$I(P_{M_1,1,i}, P_{M_1,2,j}, P_{M_1,3,k}) = \begin{cases} 1, \text{if } P_{M_1,1,i} < P_{M_1,2,j} < P_{M_1,3,k} \\ \frac{1}{2}, \text{if } P_{M_1,1,i} = P_{M_1,2,j} < P_{M_1,3,k} \text{ or } P_{M_1,1,i} < P_{M_1,2,j} = P_{M_1,3,k} \\ \frac{1}{6}, \text{if } P_{M_1,1,i} = P_{M_1,2,j} = P_{M_1,3,k} \\ 0, \text{otherwise} \end{cases},$$

$$\widehat{\text{VUS}} \text{ based on } M_2: \hat{\theta}_{M_2,o} = \frac{1}{mnl}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l} I(P_{M_2,1,i}, P_{M_2,2,j}, P_{M_2,3,k}),$$

$$I(P_{M_2,1,i}, P_{M_2,2,j}, P_{M_2,3,k}) = \begin{cases} 1, \text{if } P_{M_2,1,i} < P_{M_2,2,j} < P_{M_2,3,k} \\ \frac{1}{2}, \text{if } P_{M_2,1,i} = P_{M_2,2,j} < P_{M_2,3,k}\ or\ P_{M_2,1,i} < P_{M_2,2,j} = P_{M_2,3,k} \\ \frac{1}{6}, \text{if } P_{M_2,1,i} = P_{M_2,2,j} = P_{M_2,3,k} \\ 0, \text{otherwise,} \end{cases},$$

where, $I(P_{M_1,1,i}, P_{M_1,2,j}, P_{M_1,3,k})$ and $I(P_{M_2,1,i}, P_{M_2,2,j}, P_{M_2,3,k})$ are indicator function adjusted for ties. The indicator function gives a score by comparing predicted probability in randomly selected subject from $Y = 1$, predicted probability from $Y = 2$, and predicted probability from $Y = 3$. If the predicted probability in randomly selected subjects from $Y = 3$ is larger than the one from $Y = 2$ and the predicted probability in randomly selected subject from $Y = 2$ is larger than the one from $Y = 1$, the indicator function gives a higher score. If the order of the predicted probability for severity is correct, the indicator function generates 1/2 when a tie is apparent and 1/6 when two ties are apparent. $\hat{\theta}_{M_1,o} = 1$ and $\hat{\theta}_{M_2,o} = 1$ mean perfect order, while $\hat{\theta}_{M_1,o} = 0$ and $\hat{\theta}_{M_2,o} = 0$ mean perfectly unordered.

Expected values of $\hat{\theta}_{o1}$ and $\hat{\theta}_{o2}$ are

$$E(\hat{\theta}_{M_1,o}) = P(\,P_{M_1,1,i} < P_{M_1,2,j} < P_{M_1,3,k}) + \frac{1}{2}P(P_{M_1,1,i} < P_{M_1,2,j} = P_{M_1,3,k})$$
$$+\frac{1}{2}P(P_{M_1,1,i} = P_{M_1,2,j} < P_{M_1,3,k}) + \frac{1}{6}P(P_{M_1,1,i} = P_{M_1,2,j} = P_{M_1,3,k}),$$
$$E(\hat{\theta}_{M_2,o}) = P(P_{M_2,1,i} < P_{M_2,2,j} < P_{M_2,3,k}) + \frac{1}{2}P(P_{M_2,1,i} < P_{M_2,2,j} = P_{M_2,3,k})$$
$$+\frac{1}{2}P(P_{M_2,1,i} = P_{M_2,2,j} < P_{M_2,3,k}) + \frac{1}{6}P(P_{M_2,1,i} = P_{M_2,2,j} = P_{M_2,3,k}).$$

$Var(\hat{\theta}_{M_1,o}), Var(\hat{\theta}_{M_2,o})$ and r are used for variance estimation for ($\hat{\theta}_{M_2,o}$ - $\hat{\theta}_{M_1,o}$) in Dreiseitl et al. (2000). Variance of VUS based on M_1 is

$$Var(\hat{\theta}_{M_1,o}) = \sigma^2_{M_1,o} = \frac{1}{mnl}[\theta_{M_1,o}(1-\theta_{M_1,o}) + (l-1)(q_{M_1,12} - {\theta_{M_1,o}}^2)$$
$$+(n-1)(q_{M_1,13} - {\theta_{M_1,o}}^2) + (m-1)(q_{M_1,23} - {\theta_{M_1,o}}^2)$$
$$+(n-1)(l-1)(q_{M_1,1} - {\theta_{M_1,o}}^2)$$
$$+(m-1)(l-1)(q_{M_1,2} - {\theta_{M_1,o}}^2)$$
$$+(m-1)(n-1)(q_{M_1,3} - {\theta_{M_1,o}}^2)],$$

$$q_{M_1,12} = P\left[I(P_{M_1,1,i}, P_{M_1,2,j}, P_{M_1,3,k}) = I\left(P_{M_1,1,i}, P_{M_1,2,j}, P_{M_1,3,k'}\right) = 1\right], k \neq k'$$

$$q_{M_1,13} = P\left[I\left(P_{M_1,1,i}, P_{M_1,2,j}, P_{M_1,3,k}\right) = I\left(P_{M_1,1,i}, P_{M_1,2,j'}, P_{M_1,3,k}\right) = 1\right], j \neq j',$$

$$q_{M_1,23} = P\left[I\left(P_{M_1,1,i}, P_{M_1,2,j}, P_{M_1,3,k}\right) = I\left(P_{M_1,1,i'}, P_{M_1,2,j}, P_{M_1,3,k}\right) = 1\right], i \neq i',$$

$$q_{M_1,1} = P\left[I\left(P_{M_1,1,i}, P_{M_1,2,j}, P_{M_1,3,k}\right) = I\left(P_{M_1,1,i}, P_{M_1,2,j'}, P_{M_1,3,k'}\right) = 1\right], j \neq j', k \neq k',$$

$$q_{M_1,2} = P\left[I\left(P_{M_1,1,i}, P_{M_1,2,j}, P_{M_1,3,k}\right) = I\left(P_{M_1,1,i'}, P_{M_1,2,j}, P_{M_1,3,k'}\right) = 1\right], i \neq i', k \neq k',$$

$$q_{M_1,3} = P\left[I\left(P_{M_1,1,i}, P_{M_1,2,j}, P_{M_1,3,k}\right) = I\left(P_{M_1,1,i'}, P_{M_1,2,j'}, P_{M_1,3,k}\right) = 1\right], i \neq i', j \neq j',$$

where, $q_{M_1,12}$ is the probability of correctly scoring for three subjects $P_{M_1,1,i}$, $P_{M_1,2,j}$, $P_{M_1,3,k}$ and correctly scoring $P_{M_1,1,i}, P_{M_1,2,j}$ and different $Y = 3$ subject $P_{M_1,3,k'}$ in M_1. $q_{M_1,13}$, $q_{M_1,23}$, $q_{M_1,1}$, $q_{M_1,2}$ and $q_{M_1,3}$ have interpretations similar to $q_{M_1,12}$. Estimation for $q_{M_1,12}$, $q_{M_1,13}$, $q_{M_1,23}$, $q_{M_1,1}$, $q_{M_1,2}$ and $q_{M_1,3}$ are obtained as follow:

$$\hat{q}_{M_1,12} = \frac{1}{mnl(l-1)} \times \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l}\sum_{\substack{k=1\\k\neq k'}}^{l} I\left(P_{M_1,1,i}, P_{M_1,2,j}, P_{M_1,3,k}\right) I\left(P_{M_1,1,i}, P_{M_1,2,j}, P_{M_1,3,k'}\right),$$

$$\hat{q}_{M_1,13} = \frac{1}{mnl(n-1)} \times \sum_{i=1}^{m}\sum_{\substack{j=1\\j\neq j'}}^{n}\sum_{k=1}^{l}\sum_{k=1}^{l} I\left(P_{M_1,1,i}, P_{M_1,2,j}, P_{M_1,3,k}\right) I\left(P_{M_1,1,i}, P_{M_1,2,j'}, P_{M_1,3,k}\right),$$

$$\hat{q}_{M_1,23} = \frac{1}{mnl(m-1)} \times \sum_{\substack{i=1\\i\neq i'}}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l}\sum_{k=1}^{l} I\left(P_{M_1,1,i}, P_{M_1,2,j}, P_{M_1,3,k}\right) I\left(P_{M_1,1,i'}, P_{M_1,2,j}, P_{M_1,3,k}\right),$$

$$\hat{q}_{M_1,1} = \frac{1}{mnl(n-1)(l-1)}$$

$$\times \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l}\sum_{\substack{j=1\\ j\neq j'}}^{n}\sum_{\substack{k=1\\ k\neq k'}}^{l} I\left(P_{M_1,1,i}, P_{M_1,2,j}, P_{M_1,3,k}\right)\ I(P_{M_1,1,i}, P_{M_1,2,j'}, P_{M_1,3,k'}),$$

$$\hat{q}_{M_1,2} = \frac{1}{mnl(m-1)(l-1)}$$

$$\times \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l}\sum_{\substack{i=1\\ i\neq i'}}^{m}\sum_{\substack{k=1\\ k\neq k'}}^{l} I\left(P_{M_1,1,i}, P_{M_1,2,j}, P_{M_1,3,k}\right) I(P_{M_1,1,i'}, P_{M_1,2,j}, P_{M_1,3,k'}),$$

$$\hat{q}_{M_1,3} = \frac{1}{mnl(m-1)(n-1)}$$

$$\times \sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l}\sum_{\substack{i=1\\ i\neq i'}}^{n}\sum_{\substack{k=1\\ k\neq k'}}^{l} I\left(P_{M_1,1,i}, P_{M_1,2,j}, P_{M_1,3,k}\right) I(P_{M_1,1,i'}, P_{M_1,2,j'}, P_{M_1,3,k}).$$

Variance of $\widehat{\mathrm{VUS}}$ based on M_2 is

$$Var\left(\hat{\theta}_{M_2,o}\right) = \sigma^2_{M_2,o} = \frac{1}{mnl}[\theta_{M_2,o}\left(1-\theta_{M_2,o}\right) + (l-1)\left(q_{M_2,12} - \theta_{M_2,o}\right)$$

$$+(n-1)\left(q_{M_2,13} - {\theta_{M_2,o}}^2\right) + (m-1)\left(q_{M_2,23} - {\theta_{M_2,o}}^2\right)$$

$$+(n-1)(l-1)\left(q_{M_2,1} - {\theta_{M_2,o}}^2\right) + (m-1)(l-1)\left(q_{M_2,2} - {\theta_{M_2,o}}^2\right)$$

$$+(m-1)(n-1)\left(q_{M_2,3} - {\theta_{M_2,o}}^2\right)],$$

$$q_{M_2,12} = P\left[I\left(P_{M_2,1,i}, P_{M_2,2,j}, P_{M_2,3,k}\right) = I\left(P_{M_2,1,i}, P_{M_2,2,j}, P_{M_2,3,k'}\right) = 1\right], k \neq k',$$

$$q_{M_2,13} = P\left[I\left(P_{M_2,1,i}, P_{M_2,2,j}, P_{M_2,3,k}\right) = I\left(P_{M_2,1,i}, P_{M_2,2,j'}, P_{M_2,3,k}\right) = 1\right], j \neq j',$$

$$q_{M_2,23} = P\left[I\left(P_{M_2,1,i}, P_{M_2,2,j}, P_{M_2,3,k}\right) = I\left(P_{M_2,1,i'}, P_{M_2,2,j}, P_{M_2,3,k}\right) = 1\right], i \neq i',$$

$$q_{M_2,1} = P\left[I\left(P_{M_2,1,i}, P_{M_2,2,j}, P_{M_2,3,k}\right) = I\left(P_{M_2,1,i}, P_{M_2,2,j'}, P_{M_2,3,k'}\right) = 1\right], j \neq j', k \neq k',$$

$$q_{M_2,2} = P\left[I\left(P_{M_2,1,i}, P_{M_2,2,j}, P_{M_2,3,k}\right) = I\left(P_{M_2,1,i'}, P_{M_2,2,j}, P_{M_2,3,k'}\right) = 1\right], i \neq i', k \neq k',$$

$$q_{M_2,3} = P\left[I\left(P_{M_2,1,i}, P_{M_2,2,j}, P_{M_2,3,k}\right) = I\left(P_{M_2,1,i'}, P_{M_2,2,j'}, P_{M_2,3,k}\right) = 1\right], i \neq i', j \neq j',$$

where, $q_{M_2,12}$ is the probability of correctly scoring for three subjects $P_{M_2,1,i}$, $P_{M_2,2,j}$, $P_{M_2,3,k}$ and correctly scoring $P_{M_2,1,i}$, $P_{M_2,2,j}$ and different Y=3 subject $P_{M_2,3,k'}$ in M_1. $q_{M_2,13}$, $q_{M_2,23}$, $q_{M_2,1}$, $q_{M_2,2}$ and $q_{M_2,3}$ have interpretations similar to $q_{M_2,12}$. Estimation for $q_{M_2,12}$, $q_{M_2,13}$, $q_{M_2,23}$, $q_{M_2,1}$, $q_{M_2,2}$ and $q_{M_2,3}$ are obtained as follow:

$$\hat{q}_{M_2,12} = \frac{1}{mnl(l-1)} \times \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{l} \sum_{\substack{k=1 \\ k \neq k'}}^{l} I\left(P_{M_2,1,i}, P_{M_2,2,j}, P_{M_2,3,k}\right) I\left(P_{M_2,1,i}, P_{M_2,2,j}, P_{M_2,3,k'}\right),$$

$$\hat{q}_{M_2,13} = \frac{1}{mnl(n-1)} \times \sum_{i=1}^{m} \sum_{\substack{j=1 \\ j \neq j'}}^{n} \sum_{k=1}^{l} \sum_{k=1}^{l} I\left(P_{M_2,1,i}, P_{M_2,2,j}, P_{M_2,3,k}\right) I\left(P_{M_2,1,i}, P_{M_2,2,j'}, P_{M_2,3,k}\right),$$

$$\hat{q}_{M_2,23} = \frac{1}{mnl(m-1)} \times \sum_{\substack{i=1 \\ i \neq i'}}^{m} \sum_{j=1}^{n} \sum_{k=1}^{l} \sum_{k=1}^{l} I\left(P_{M_2,1,i}, P_{M_2,2,j}, P_{M_2,3,k}\right) I\left(P_{M_2,1,i'}, P_{M_2,2,j}, P_{M_2,3,k}\right),$$

$$\hat{q}_{M_2,1} = \frac{1}{mnl(n-1)(l-1)}$$

$$\times \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{l} \sum_{\substack{j=1 \\ j \neq j\prime}}^{n} \sum_{\substack{k=1 \\ k \neq k'}}^{l} I\big(P_{M_2,1,i}, P_{M_2,2,j}, P_{M_2,3,k}\big) I\big(P_{M_2,1,i}, P_{M_2,2,j'}, P_{M_2,3,k'}\big),$$

$$\hat{q}_{M_2,2} = \frac{1}{mnl(m-1)(l-1)}$$

$$\times \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{l} \sum_{\substack{i=1 \\ i \neq i\prime}}^{m} \sum_{\substack{k=1 \\ k \neq k'}}^{l} I\big(P_{M_2,1,i}, P_{M_2,2,j}, P_{M_2,3,k}\big) I\big(P_{M_2,1,i'}, P_{M_2,2,j}, P_{M_2,3,k'}\big),$$

$$\hat{q}_{M_2,3} = \frac{1}{mnl(m-1)(n-1)}$$

$$\times \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{l} \sum_{\substack{i=1 \\ i \neq i\prime}}^{n} \sum_{\substack{k=1 \\ k \neq k'}}^{l} I\big(P_{M_2,1,i}, P_{M_2,2,j}, P_{M_2,3,k}\big) I\big(P_{M_2,1,i'}, P_{M_2,2,j'}, P_{M_2,3,k}\big).$$

r is the ratio of $Cov(\hat{\theta}_{M_1,o}, \hat{\theta}_{M_2,o})$ and $\sigma_{M_1,o}, \sigma_{M_2,o}$, and is calculated as

$$r = \frac{Cov(\hat{\theta}_{M_1,o}, \hat{\theta}_{M_2,o})}{\sigma_{M_1,o}\sigma_{M_2,o}},$$

where,

$$Cov\big(\hat{\theta}_{M_1,o}, \hat{\theta}_{M_2,o}\big) = \frac{1}{mnl}[q_{123} - \theta_{M_1,o}\theta_{M_2,o} + (l-1)\big(q_{12} - \theta_{M_1,o}\theta_{M_2,o}\big)$$

$$+(n-1)\big(q_{13} - \theta_{M_1,o}\theta_{M_2,o}\big) + (m-1)\big(q_{23} - \theta_{M_1,o}\theta_{M_2,o}\big)$$

$$+(n-1)(l-1)\big(q_1 - \theta_{M_1,o}\theta_{M_2,o}\big)$$

$$+(m-1)(l-1)\big(q_2 - \theta_{M_1,o}\theta_{M_2,o}\big)$$

$$+(m-1)(n-1)\left(q_3-\theta_{M_1,o}\theta_{M_2,o}\right)],$$

$$q_{123}=P\left[I\left(P_{M_1,1,i},P_{M_1,2,j},P_{M_1,3,k}\right)=I\left(P_{M_2,1,i},P_{M_2,2,j},P_{M_2,3,k}\right)=1\right],$$

$$q_{12}=P\left[I\left(P_{M_1,1,i},P_{M_1,2,j},P_{M_1,3,k}\right)=I\left(P_{M_2,1,i},P_{M_2,2,j},P_{M_2,3,k'}\right)=1\right],k\neq k',$$

$$q_{13}=P\left[I\left(P_{M_1,1,i},P_{M_1,2,j},P_{M_1,3,k}\right)=I\left(P_{M_2,1,i},P_{M_2,2,j'},P_{M_2,3,k}\right)=1\right],j\neq j',$$

$$q_{23}=P\left[I\left(P_{M_1,1,i},P_{M_1,2,j},P_{M_1,3,k}\right)=I\left(P_{M_2,1,i'},P_{M_2,2,j},P_{M_2,3,k}\right)=1\right],i\neq i',$$

$$q_{1}=P\left[I\left(P_{M_1,1,i},P_{M_1,2,j},P_{M_1,3,k}\right)=I\left(P_{M_2,1,i},P_{M_2,2,j'},P_{M_2,3,k'}\right)=1\right],j\neq j',k\neq k',$$

$$q_{2}=P\left[I\left(P_{M_1,1,i},P_{M_1,2,j},P_{M_1,3,k}\right)=I\left(P_{M_2,1,i'},P_{M_2,2,j},P_{M_2,3,k'}\right)=1\right],i\neq i',k\neq k',$$

$$q_{3}=P\left[I\left(P_{M_1,1,i},P_{M_1,2,j},P_{M_1,3,k}\right)=I\left(P_{M_2,1,i'},P_{M_2,2,j'},P_{M_2,3,k}\right)=1\right],i\neq i',j\neq j',$$

where, q_{123} is the probability of correctly scoring for three subjects $P_{M_1,1,i},P_{M_1,2,j},P_{M_1,3,k}$ in M_1 and correctly scoring $P_{M_2,1,i},P_{M_2,2,j},P_{M_2,3,k}$ in M_2. q_{12} the probability of correctly scoring for three subjects $P_{M_1,1,i},P_{M_1,2,j},P_{M_1,3,k}$ in M_1 and correctly scoring $P_{M_2,1,i},P_{M_2,2,j}$ and different $Y=3$ subject $P_{M_2,3,k'}$ in M_2. q_{12}, q_{13}, q_{23}, q_1, q_2, and q_3 have interpretations similar to q_{123}. Estimation for q_{123}, q_{12}, q_{13}, q_{23}, q_1, q_2, and q_3 are obtained as follows:

$$\hat{q}_{123}=\frac{1}{mnl}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l}I\left(P_{M_1,1,i},P_{M_1,2,j},P_{M_1,3,k}\right)I\left(P_{M_2,1,i},P_{M_2,2,j},P_{M_2,3,k}\right),$$

$$\hat{q}_{12}=\frac{1}{mnl(l-1)}\sum_{i=1}^{m}\sum_{j=1}^{n}\sum_{k=1}^{l}\sum_{\substack{K=1\\k\neq k\prime}}^{l}I\left(P_{M_1,1,i},P_{M_1,2,j},P_{M_1,3,k}\right)I(P_{M_2,1,i},P_{M_2,2,j},P_{M_2,3,k'}),$$

$$\hat{q}_{13} = \frac{1}{mnl(n-1)} \sum_{i=1}^{m} \sum_{\substack{j=1 \\ j \neq j'}}^{n} \sum_{k=1}^{l} \sum_{k=1}^{l} I\left(P_{M_1,1,i}, P_{M_1,2,j}, P_{M_1,3,k}\right) I\left(P_{M_2,1,i}, P_{M_2,2,j'}, P_{M_2,3,k}\right),$$

$$\hat{q}_{23} = \frac{1}{mnl(m-1)} \sum_{\substack{i=1 \\ i \neq i'}}^{m} \sum_{j=1}^{n} \sum_{k=1}^{l} \sum_{k=1}^{l} I\left(P_{M_1,1,i}, P_{M_1,2,j}, P_{M_1,3,k}\right) I\left(P_{M_2,1,i'}, P_{M_2,2,j}, P_{M_2,3,k}\right),$$

$$\hat{q}_1 = \frac{1}{mnl(n-1)(l-1)}$$

$$\times \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{l} \sum_{\substack{j=1 \\ j \neq j'}}^{n} \sum_{\substack{k=1 \\ k \neq k'}}^{l} I\left(P_{M_1,1,i}, P_{M_1,2,j}, P_{M_1,3,k}\right) I\left(P_{M_2,1,i}, P_{M_2,2,j'}, P_{M_2,3,k'}\right),$$

$$\hat{q}_2 = \frac{1}{mnl(m-1)(l-1)}$$

$$\times \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{l} \sum_{\substack{i=1 \\ i \neq i'}}^{m} \sum_{\substack{k=1 \\ k \neq k'}}^{l} I\left(P_{M_1,1,i}, P_{M_1,2,j}, P_{M_1,3,k}\right) I\left(P_{M_2,1,i'}, P_{M_2,2,j}, P_{M_2,3,k'}\right),$$

$$\hat{q}_3 = \frac{1}{mnl(m-1)(n-1)}$$

$$\times \sum_{i=1}^{m} \sum_{j=1}^{n} \sum_{k=1}^{l} \sum_{\substack{i=1 \\ i \neq i'}}^{n} \sum_{\substack{k=1 \\ k \neq k'}}^{l} I\left(P_{M_1,1,i}, P_{M_1,2,j}, P_{M_1,3,k}\right) I\left(P_{M_2,1,i'}, P_{M_2,2,j'}, P_{M_2,3,k}\right).$$

$Z_{\Delta VUS}$ is the test statistic for testing the null hypothesis, which is $\Delta VUS = 0$, as follows:

$$Z_{\Delta VUS} = \frac{\hat{\theta}_{M_2,o} - \hat{\theta}_{M_1,o}}{\sqrt{\hat{\sigma}^2_{M_1,o} + \hat{\sigma}^2_{M_2,o} - 2\hat{r}\hat{\sigma}^2_{M_1,o}\hat{\sigma}^2_{M_2,o}}},$$

and compared to a standard normal distribution. Alternatively, the bootstrapping (Efron et al., 1994) can be used to test the null hypothesis. (Nakas et al., 2004).

2.2. Net reclassification index

2.2.1 Binary outcomes

To assess the added predictive ability of a new factor for binary outcomes, the notations M_1 and M_2 are used with the same meaning as that used in 2.1.1 Binary outcomes (Agresti, 2007).

Each model's predicted probabilities are classified into two categories by arbitrary cut-off points. NRI is based on upward and downward movement of categories classified with predicted probabilities among an event group and non-event group. NRI evaluates the 'net' number of subjects reclassified correctly using M_2 over M_1. This is done by calculating how many subjects experiencing an event in the event group and how many individuals not experiencing an event decreased in the non-event group. (Pencina et al.. 2008; Kennedy et al., 2011).

NRI for binary outcomes is defined as

$$NRI_b = [P(\text{down}|Y = 1) - P(\text{up}|Y = 1)] + [P(\text{up}|Y = 2) - P(\text{down}|Y = 2)],$$

where $Y = 1$ refers to the non-event group, and $Y = 2$ means the event group. A reclassification table for binary outcome by M_1 and M_2 is shown in Table 1.

Table 1. Reclassification table for binary outcomes by M_1 and M_2

$Y = 1$

$M_1 \setminus M_2$	1	2	
1	a_1	a_2	
2	a_3	a_4	

$Y = 2$

$M_1 \setminus M_2$	1	2	
1	b_1	b_2	
2	b_3	b_4	

To estimate NRI_b, we define estimators for the four probabilities

$$\hat{P}(\text{down}|Y=1) = \hat{p}_{down,non-\,events} = \frac{\#\text{ non} - \text{events moving down}}{\#\text{ non} - \text{events}} = \frac{a_3}{a_1 + a_2 + a_3 + a_4},$$

$$\hat{P}(\text{up}|Y=1) = \hat{p}_{up,non-\,events} = \frac{\#\text{ non} - \text{events moving up}}{\#\text{ non} - \text{events}} = \frac{a_2}{a_1 + a_2 + a_3 + a_4},$$

$$\hat{P}(\text{down}|Y=2) = \hat{p}_{down,events} = \frac{\#\text{ events moving down}}{\#\text{ events}} = \frac{b_3}{b_1 + b_2 + b_3 + b_4},$$

$$\hat{P}(\text{up}|Y=2) = \hat{p}_{up,events} = \frac{\#\text{ events moving up}}{\#\text{ events}} = \frac{b_2}{b_1 + b_2 + b_3 + b_4}.$$

Using four probabilities, we define $\widehat{NRI}_b$ as follows:

$$\widehat{NRI}_b\colon\ \hat{\theta}_b = \left(\hat{p}_{down,non-\,events} - \hat{p}_{up,non-\,events}\right) + \left(\hat{p}_{up,events} - \hat{p}_{down,events}\right)$$

$$= \frac{a_3 - a_2}{a_1 + a_2 + a_3 + a_4} + \frac{b_2 - b_3}{b_1 + b_2 + b_3 + b_4}.$$

Assuming independence between event and non-event individuals and following McNemar's logic for significance testing in correlated proportions (Pencina et al., 2008), one can derive a simple asymptotic test for the null hypothesis of NRI=0. The estimation SE of $\widehat{NRI}_b$ is

$$\widehat{SE}_b: \hat{\sigma}_b = \sqrt{\frac{\hat{p}_{down,non-\ events} + \hat{p}_{up,non-events}}{\#\ non - events} + \frac{\hat{p}_{up,events} + \hat{p}_{down,events}}{\#\ events}}.$$

Z_{NRI_b} is the test statistic for testing the null hypothesis, which is $NRI_b = 0$, and it is calculated as follows:

$$Z_{NRI_b} = \frac{\hat{\theta}_b}{\hat{\sigma}_b},$$

and compared to a standard normal distribution.

2.2.2 Multiclass outcomes

To assess the added predictive ability of a new factor for binary outcomes, the notations M_1 and M_2 are used with the same meaning as that used in 2.1.2 Ordinal outcomes. $\mathbf{p}(M_1)$ and $\mathbf{p}(M_2)$ are probability vectors which are expressed as $\mathbf{p}(M_1) = \left(p_1(M_1), \cdots, p_c(M_1)\right)'$, and $\mathbf{p}(M_2) = \left(p_1(M_2), \cdots, p_c(M_2)\right)'$. If the predicted probability of M_1 is the greatest among the three categories, $p_c(M_1) = \max(\mathbf{p}(M_1)|Y = c)$, then the predicted category based on M_1 is defined as c. Similarly, if the equation is written as $p_c(M_2) = \max(\mathbf{p}(M_2)|Y = c)$, then the predicted category based on M_2 is defined as c.

Reclassification index (RI) for multiclass outcomes refers to the difference between correct classification of the probability of M_2 and correct classification of the probability of M_1 (Li et al., 2013). RI for multiclass outcomes is

$$RI_m = \sum_{c=1}^{C} w_c[\,P\{p_c(M_2) = \max \mathbf{p}(M_2)|Y = c\} - P\{p_c(M_1) = \max \mathbf{p}(M_1)|Y = c\}]$$

$$, c=1,\cdots,C.$$

NRI for multiclass outcomes means the probability that added factors in M_2 lead to correct classification of subjects who are incorrectly classified using the smaller model M_1 (Li et al., 2013). NRI for multiclass outcomes is

Table 2. Reclassification table for multiclass outcomes by M_1 and M_2

Y=1

$M_1 \setminus M_2$	1	2	3
1	a_1	a_2	a_3
2	a_4	a_5	a_6
3	a_7	a_8	a_9

$M_1 \setminus M_2$	1	2	3
1	a_1	a_2	a_3
2	a_4	a_5	a_6
3	a_7	a_8	a_9

Y=2

$M_1 \setminus M_2$	1	2	3
1	b_1	b_2	b_3
2	b_4	b_5	b_6
3	b_7	b_8	b_9

$M_1 \setminus M_2$	1	2	3
1	b_1	b_2	b_3
2	b_4	b_5	b_6
3	b_7	b_8	b_9

Y=3

$M_1 \setminus M_2$	1	2	3
1	c_1	c_2	c_3
2	c_4	c_5	c_6
3	c_7	c_8	c_9

$M_1 \setminus M_2$	1	2	3
1	c_1	c_2	c_3
2	c_4	c_5	c_6
3	c_7	c_8	c_9

$$NRI_m = \sum_{c=1}^{C} w_c \, P\{p_c(M_2) = \max \mathbf{p}(M_2), p_c(M_1) \neq \max \mathbf{p}(M_1) | Y = c\},$$

where w_c is positive weight for the c^{th} category. A reclassification table for multiclass outcomes by M_1 and M_2 is shown in Table 2.

Estimated RI and variance for multiclass outcomes are

$$\widehat{RI}_m\text{: } \hat{\theta}_{m_1} = \sum_{c=1}^{C} \frac{w_c}{n_c} \sum_{i=1}^{n} \{I(\hat{p}_{ci}(M_2) = \max \widehat{\mathbf{p}}_i(M_2), Y_i = c)$$

$$-I(\hat{p}_{ci}(M_1) = \max \widehat{\mathbf{p}}_i(M_1), Y_i = c\}$$

$$= \frac{1}{n_1}\left(\frac{(a_4 + a_7) - (a_2 + a_3)}{a_1 + \cdots + a_9}\right) + \frac{1}{n_2}\left(\frac{(b_2 + b_8) - (b_4 + b_6)}{b_1 + \cdots + b_9}\right)$$

$$+ \frac{1}{n_3}\left(\frac{(c_3 + c_6) - (c_7 + c_8)}{c_1 + \cdots + c_9}\right),$$

$$Var(\hat{\theta}_{m_1}) = \hat{\sigma}^2_{m_1} = \sum_{c=1}^{C} \frac{w_c^2}{\rho_c}(a_c + b_c + 2e_c) - \sum_{i=1}^{C}\sum_{j=1}^{C} w_i w_j (a_i - b_i)(a_j - b_j),$$

where $a_c = \mathrm{P}\{p_c(M_1) = \max \mathbf{p}(M_1) | Y = c\}$, $b_c = \mathrm{P}\{p_c(M_2) = \max \mathbf{p}(M_2) | Y = c\}$, $e_c = \mathrm{P}\{p_c(M_1) = \max \mathbf{p}(M_1) | Y = c,\ p_c(M_2) = \max \mathbf{p}(M_2) | Y = c\}$, and ρ_c is the prevalence for the c^{th} category.

Estimated NRI and variance for multiclass are

$$\widehat{NRI}_m\text{: } \hat{\theta}_{m_2} = \sum_{c=1}^{C} \frac{w_c}{n_c} \sum_{i=1}^{n} I\{\hat{p}_{ci}(M_2) = \max \widehat{\mathbf{p}}_i(M_2), \hat{p}_{ci}(M_1) \neq \max \widehat{\mathbf{p}}_i(M_1), Y = c\}$$

$$= \frac{1}{n_1}\left(\frac{a_4 + a_7}{a_1 + \cdots + a_9}\right) + \frac{1}{n_2}\left(\frac{b_2 + b_8}{b_1 + \cdots + b_9}\right) + \frac{1}{n_3}\left(\frac{c_3 + c_6}{c_1 + \cdots + c_9}\right),$$

$$Var(\hat{\theta}_{m_2}) = \hat{\sigma}^2_{m_2} = \sum_{c=1}^{C} \frac{w_c^2}{\rho_c} \hat{d}_c - \sum_{i=1}^{C} \sum_{j=1}^{C} w_i w_j \, \hat{d}_i \hat{d}_j,$$

where, $\hat{d}_c = P(\, p_c(M_2) = \max \mathbf{p}(M_2), p_c(M_1) \neq \max \mathbf{p}(M_1) | Y = c)$.

Li (2013) derived a simple asymptotic test for the null hypothesis of RI=0 and NRI=0. Z_{RI_m} and Z_{NRI_m} are the test statistics for testing the null hypothesis which is $RI_m = 0$ and $NRI_m = 0$, and they are calculated as

$$Z_{RI_m} = \frac{\hat{\theta}_{m_1}}{\sqrt{\hat{\sigma}^2_{m_1}}}$$

and

$$Z_{NRI_m} = \frac{\hat{\theta}_{m_2}}{\sqrt{\hat{\sigma}^2_{m_2}}}.$$

These values are compared to a standard normal distribution.

III. Proposed method

3.1. Net reclassification index for ordinal outcomes

To assess the added predictive ability of a new factor for binary outcomes, the notations M_1 and M_2 are used with the same meaning as that used in 2.1.2 Ordinal outcomes.

Consider the case that the number of outcome categories is three. If the predicted probability of M_1 is the greatest among the three categories, $p_c(M_1) = \max(\mathbf{p}(M_1)|Y = c)$, then the predicted category based on M_1 is defined as c. Similarly, if the equation is written as $p_c(M_2) = \max(\mathbf{p}(M_2)|Y = c)$, then the predicted category based on M_2 is defined as c.

For a better understanding, reclassification tables with weights for ordinal outcome by M_1 and M_2 are provided in the Table 3, Table 4, and Table 5.

Herein, applying NRI for ordinal outcomes is proposed by extending NRI for binary outcomes (Pencina et al., 2008) and NRI for multiclass outcomes (Dreiseitl et al., 2000; Nakas et al., 2004). NRI for ordinal categories is defined based on a predicted category. Additionally, weights that consider closeness to the true category are taken into account when counting reclassification.

NRI for ordinal outcomes is defined as

$$NRI_o = \sum_{c=1}^{C} w[P\{(|C(M_1) - c| > |C(M_2) - c|)|Y = c\}$$
$$- P\{(|C(M_1) - c| < |C(M_2) - c|)|Y = c\}],$$

where c is the true outcome ($c = 1, \cdots, C$), $C(M_1)$ is the predicted category based on predicted probability in M_1 ($C(M_1) = 1, \cdots, C$), $C(M_2)$ is the predicted category based on predicted probability in M_2 ($C(M_2) = 1, \cdots, C$), and w is the weight function of c, $C(M_1)$, and $C(M_2)$.

We consider three different weights, w_1, w_2 and w_3,

$$w_1 = \frac{|C(M_1) - c| - |C(M_2) - c|}{C - 1},$$

$$w_2 = \begin{cases} \dfrac{|C(M_1) - c| - |C(M_2) - c| - 1}{C - 1}, & \text{if } C(M_1) = c, C(M_2) \neq c, \\ \dfrac{|C(M_1) - c| - |C(M_2) - c| + 1}{C - 1}, & \text{if } C(M_2) = c, C(M_1) \neq c, \\ \dfrac{|C(M_1) - c| - |C(M_2) - c|}{C - 1}, & \text{otherwise,} \end{cases}$$

$$w_3 = \begin{cases} -1, & \text{if } C(M_1) = c, C(M_2) \neq c, \\ 1, & \text{if } C(M_2) = c, C(M_1) \neq c, \\ \dfrac{|C(M_1) - c| - |C(M_2) - c|}{C - 1}, & \text{otherwise.} \end{cases}$$

That is, w_1 considers only closeness from the true category; meanwhile, w_2 and w_3 not only consider closeness from the true category but also the correct classification for $C(M_1)$ and $C(M_2)$ from the true category. For w_1, the closer the distance between $C(M_2)$ and the true category, the higher the weight that is given. For w_2, high weight, greater than 1, is given when $C(M_2)$ and the true category are the same and when $C(M_2)$ and the true category get closer. Otherwise, when only closeness from the true category is taken into account, weight below 1 is given. For w_3, when $C(M_3)$ and the true category are the same, weight is given as 1. When $C(M_3)$ is just close to the true category, the weight less than 1.

Estimated NRI for ordinal outcomes is written below,

$$\widehat{NRI}_o : \hat{\theta}_o = \sum_{c=1}^{C} \sum_{C(M_1)=1}^{C} \sum_{C(M_2)=1}^{C} \frac{w_{cC(M_1)C(M_2)} n_{cC(M_1)C(M_2)}}{n_{++c}},$$

where $n_{cC(M_1)C(M_2)}$ is the frequency classified by $C(M_1)$ and $C(M_2)$ given $Y = c$, and n_{++c} is the sample size for the c^{th} category. $\widehat{NRI}_o$ represents the three methods $\widehat{NRI}_{o,w_1}$, $\widehat{NRI}_{o,w_2}$ and $\widehat{NRI}_{o,w_3}$ according to weight.

Table 3. Reclassification table with weight w_1 for ordinal outcomes by M_1 and M_2

$n_{C(M_1)C(M_2)c}$ $w_{C(M_1)C(M_2)c}$

Non-event

$Y = 1$

$M_1 \setminus M_2$	1	2	3	
1	p_{111} (n_{111})	p_{121} (n_{121})	p_{131} (n_{131})	p_{1+1} (n_{1+1})
2	p_{211} (n_{211})	p_{221} (n_{221})	p_{231} (n_{231})	p_{2+1} (n_{2+1})
3	p_{311} (n_{311})	p_{321} (n_{321})	p_{331} (n_{331})	p_{3+1} (n_{3+1})
	p_{+11} (n_{+11})	p_{+21} (n_{+21})	p_{+31} (n_{+31})	p_{++1} (n_{++1})

	1	2	3	
1		-1/2	-2/2	
2	1/2		-1/2	
3	2/2	1/2		

$Y = 2$

↓

$M_1 \setminus M_2$	1	2	3	
1	p_{112} (n_{112})	p_{122} (n_{122})	p_{132} (n_{132})	p_{1+2} (n_{1+2})
2	p_{212} (n_{212})	p_{222} (n_{222})	p_{232} (n_{232})	p_{2+2} (n_{2+2})
3	p_{312} (n_{312})	p_{322} (n_{322})	p_{332} (n_{332})	p_{3+2} (n_{3+2})
	p_{+12} (n_{+12})	p_{+22} (n_{+22})	p_{+32} (n_{+32})	p_{++2} (n_{++2})

	1	2	3	
1		1/2		
2	-1/2		-1/2	
3		1/2		

$Y = 3$

Event

$M_1 \setminus M_2$	1	2	3	
1	p_{113} (n_{113})	p_{123} (n_{123})	p_{133} (n_{133})	p_{1+3} (n_{1+3})
2	p_{213} (n_{213})	p_{223} (n_{223})	p_{233} (n_{233})	p_{2+3} (n_{2+3})
3	p_{313} (n_{313})	p_{323} (n_{323})	p_{333} (n_{333})	p_{3+3} (n_{3+3})
	p_{+13} (n_{+13})	p_{+23} (n_{+23})	p_{+33} (n_{+33})	p_{++3} (n_{++3})

	1	2	3	
1		1/2	2/2	
2	-1/2		1/2	
3	-2/2	-1/2		

Table 4. Reclassification table with weight w_2 for ordinal outcomes by M_1 and M_2

$n_{C(M_1)C(M_2)c}$ $w_{C(M_1)C(M_2)c}$

Non-event

$Y=1$

$M_1 \setminus M_2$	1	2	3	
1	p_{111} (n_{111})	p_{121} (n_{121})	p_{131} (n_{131})	p_{1+1} (n_{1+1})
2	p_{211} (n_{211})	p_{221} (n_{221})	p_{231} (n_{231})	p_{2+1} (n_{2+1})
3	p_{311} (n_{311})	p_{321} (n_{321})	p_{331} (n_{331})	p_{3+1} (n_{3+1})
	p_{+11} (n_{+11})	p_{+21} (n_{+21})	p_{+31} (n_{+31})	p_{++1} (n_{++1})

	1	2	3	
1		-1	-3/2	
2	1		-1/2	
3	3/2	1/2		

↓

$Y=2$

$M_1 \setminus M_2$	1	2	3	
1	p_{112} (n_{112})	p_{122} (n_{122})	p_{132} (n_{132})	p_{1+2} (n_{1+2})
2	p_{212} (n_{212})	p_{222} (n_{222})	p_{232} (n_{232})	p_{2+2} (n_{2+2})
3	p_{312} (n_{312})	p_{322} (n_{322})	p_{332} (n_{332})	p_{3+2} (n_{3+2})
	p_{+12} (n_{+12})	p_{+22} (n_{+22})	p_{+32} (n_{+32})	p_{++2} (n_{++2})

	1	2	3	
1		1		
2	-1		-1	
3		1		

Event

$Y=3$

$M_1 \setminus M_2$	1	2	3	
1	p_{113} (n_{113})	p_{123} (n_{123})	p_{133} (n_{133})	p_{1+3} (n_{1+3})
2	p_{213} (n_{213})	p_{223} (n_{223})	p_{233} (n_{233})	p_{2+3} (n_{2+3})
3	p_{313} (n_{313})	p_{323} (n_{323})	p_{333} (n_{333})	p_{3+3} (n_{3+3})
	p_{+13} (n_{+13})	p_{+23} (n_{+23})	p_{+33} (n_{+33})	p_{++3} (n_{++3})

	1	2	3	
1		1/2	3/2	
2	-1/2		1	
3	-3/2	-1		

Table 5. Reclassification table with weight w_3 for ordinal outcomes by M_1 and M_2

$n_{C(M_1)C(M_2)c}$ $w_{C(M_1)C(M_2)c}$

Non-event

$Y = 1$

$M_1 \setminus M_2$	1	2	3	
1	p_{111} (n_{111})	p_{121} (n_{121})	p_{131} (n_{131})	p_{1+1} (n_{1+1})
2	p_{211} (n_{211})	p_{221} (n_{221})	p_{231} (n_{231})	p_{2+1} (n_{2+1})
3	p_{311} (n_{311})	p_{321} (n_{321})	p_{331} (n_{331})	p_{3+1} (n_{3+1})
	p_{+11} (n_{+11})	p_{+21} (n_{+21})	p_{+31} (n_{+31})	p_{++1} (n_{++1})

	1	2	3	
1		-1	-1	
2	1		-1/2	
3	1	1/2		

↓

$Y = 2$

$M_1 \setminus M_2$	1	2	3	
1	p_{112} (n_{112})	p_{122} (n_{122})	p_{132} (n_{132})	p_{1+2} (n_{1+2})
2	p_{212} (n_{212})	p_{222} (n_{222})	p_{232} (n_{232})	p_{2+2} (n_{2+2})
3	p_{312} (n_{312})	p_{322} (n_{322})	p_{332} (n_{332})	p_{3+2} (n_{3+2})
	p_{+12} (n_{+12})	p_{+22} (n_{+22})	p_{+32} (n_{+32})	p_{++2} (n_{++2})

	1	2	3	
1		1		
2	-1		-1	
3		1		

Event

$Y = 3$

$M_1 \setminus M_2$	1	2	3	
1	p_{113} (n_{113})	p_{123} (n_{123})	p_{133} (n_{133})	p_{1+3} (n_{1+3})
2	p_{213} (n_{213})	p_{223} (n_{223})	p_{233} (n_{233})	p_{2+3} (n_{2+3})
3	p_{313} (n_{313})	p_{323} (n_{323})	p_{333} (n_{333})	p_{3+3} (n_{3+3})
	p_{+13} (n_{+13})	p_{+23} (n_{+23})	p_{+33} (n_{+33})	p_{++3} (n_{++3})

	1	2	3	
1		1/2	1	
2	-1/2		1	
3	-1	-1		

3.2. Standard error of net reclassification index for ordinal outcomes

For estimating the standard error of NRI for ordinal outcomes, a different form of $\widehat{NRI}_o$ from the one suggested in 3.1 is defined. It is expressed with the product of $\boldsymbol{a}$ vector and $\boldsymbol{b}$ vector expanded from McNemar's test (Stuart, 1955; Maxwell, 1970; Sun et al., 2008) as follows:

$$\widehat{NRI}_o: \hat{\theta}_o = \sum_{c=1}^{C} \hat{\theta}_{o,c} = \sum_{c=1}^{C} \boldsymbol{a}_c' \boldsymbol{b}_c.$$

As an example, when the number of outcome categories equals three and the weight is w_1, $\boldsymbol{a}_c, \boldsymbol{b}_c$ for $c = 1,2,3$ is defined as follows,

$$\boldsymbol{a}_1 = \begin{pmatrix} 2/2 \\ 1/2 \end{pmatrix}, \boldsymbol{b}_1 = \begin{pmatrix} p_{1+1} - p_{11+} \\ p_{1+2} - p_{12+} \end{pmatrix},$$

$$\boldsymbol{a}_2 = (1/2), \boldsymbol{b}_2 = (p_{2+2} - p_{22+}),$$

$$\boldsymbol{a}_3 = \begin{pmatrix} 2/2 \\ 1/2 \end{pmatrix}, \boldsymbol{b}_3 = \begin{pmatrix} p_{3+3} - p_{33+} \\ p_{3+2} - p_{32+} \end{pmatrix}.$$

.

Estimation of NRI for each outcome category is obtained as follows,

$$\hat{\theta}_{o,1} = \boldsymbol{a}_1' \boldsymbol{b}_1 = \begin{pmatrix} \frac{2}{2} & \frac{1}{2} \end{pmatrix} \begin{pmatrix} p_{1+1} - p_{11+} \\ p_{1+2} - p_{12+} \end{pmatrix} = \frac{2}{2}(p_{1+1} - p_{11+}) + \frac{1}{2}(p_{1+2} - p_{12+})$$

$$= \frac{1}{2}p_{121} + \frac{2}{2}p_{131} - \frac{1}{2}p_{112} + \frac{1}{2}p_{132} - \frac{2}{2}p_{113} - \frac{1}{2}p_{123},$$

$$\hat{\theta}_{o,2} = \boldsymbol{a}_2'\boldsymbol{b}_2 = \frac{1}{2}(p_{2+2} + p_{22+}) = \left(\frac{1}{2}\right)(p_{2+2} - p_{22+})$$

$$= -\frac{1}{2}p_{221} + \frac{1}{2}p_{212} + \frac{1}{2}p_{232} - \frac{1}{2}p_{223},$$

$$\hat{\theta}_{o,3} = \boldsymbol{a}_3'\boldsymbol{b}_3 = \begin{pmatrix} 2/2 \\ 1/2 \end{pmatrix}\begin{pmatrix} p_{3+3} - p_{33+} \\ p_{3+2} - p_{32+} \end{pmatrix} = \frac{2}{2}(p_{3+3} - p_{33+}) + \frac{1}{2}(p_{3+2} - p_{32+})$$

$$= -\frac{1}{2}p_{321} - \frac{2}{2}p_{331} + \frac{1}{2}p_{312} - \frac{1}{2}p_{332} + \frac{2}{2}p_{313} + \frac{1}{2}p_{323}.$$

The standard error of the proposed NRI can be estimated utilizing the variance estimation procedures of Stuart-Maxwell test (generalized McNemar's test) and Bhapkar's test statistics.

$$Var(\hat{\theta}_o) = \hat{\sigma}_o^2 = \sum_{c=1}^{C} Var(\hat{\theta}_{o,c}) = Var(\hat{\theta}_{o,1}) + Var(\hat{\theta}_{o,2}) + Var(\hat{\theta}_{o,3})$$

$$= \frac{1}{n_{++1}}\boldsymbol{a}_1'V_{o,1}\boldsymbol{a}_1 + \frac{1}{n_{++2}}\boldsymbol{a}_2'V_{o,2}\boldsymbol{a}_2 + \frac{1}{n_{++3}}\boldsymbol{a}_3'V_{o,3}\boldsymbol{a}_3$$

Variance-covariance matrix of the vectors is $V_{o,c}$, and it is $(C-1)\times(C-1)$ or $(C-2)\times(C-2)$ dimension. The elements of $\hat{V}_{o,c}$ are obtained from two methods: First is the Stuart-Maxwell test (Generalized McNemar) written below:

$$\hat{v}_{o,cC(M_1)C(M_2)} = -\left(p_{cC(M_1)C(M_2)} + p_{cC(M_2)C(M_1)}\right)$$

$$\text{, for } C(M_1) \neq C(M_2) \text{ and } C(M_1), C(M_2) = 1, \cdots, C,$$

$$\hat{v}_{o,cC(M_1)C(M_2)} = p_{cC(M_1)+} + p_{c,+C(M_2)} - 2p_{cC(M_1)C(M_2)},$$

$$\text{, for } C(M_1) = C(M_2) \text{ and } C(M_1), C(M_2) = 1, \cdots, C,$$

where c means a true category, and $C(M_1)$ and $C(M_2)$ mean selected columns and rows. $\hat{V}_{o,1}, \hat{V}_{o,2}$ and $\hat{V}_{o,3}$ for $c = 1,2,3$ is defined as follows:

$$\hat{V}_{o,1} = \begin{bmatrix} p_{1,1+} + p_{1,+1} - 2p_{1,11} & -(p_{1,12} + p_{1,21}) \\ -(p_{1,21} + p_{1,12}) & p_{1,2+} + p_{1,+2} - 2p_{1,22} \end{bmatrix},$$

$$\hat{V}_{o,2} = [p_{2,2+} + p_{2,+2} - 2p_{2,22}],$$

$$\hat{V}_{o,3} = \begin{bmatrix} \pi_{3,3+} + \pi_{3,+3} - 2\pi_{3,33} & -(\pi_{3,32} + \pi_{3,23}) \\ -(\pi_{3,23} + \pi_{3,32}) & \pi_{3,2+} + \pi_{3,+2} - 2\pi_{3,22} \end{bmatrix}.$$

Second is the Bhapkar's test:

$$\hat{v}_{o,cC(M_1)C(M_2)} = -(p_{cC(M_1)C(M_2)} + p_{cC(M_2)C(M_1)})$$
$$-(p_{c,+C(M_1)} - p_{c,C(M_1)+})(p_{c,+C(M_2)} - p_{c,C(M_2)+})$$
$$\text{, for } C(M_1) \neq C(M_2) \text{ and } C(M_1), C(M_2) = 1, \cdots, C,$$

$$\hat{v}_{o,cC(M_1)C(M_2)} = p_{cC(M_1)+} + p_{c,+C(M_2)} - 2p_{cC(M_1)C(M_2)} - \left(p_{c,+C(M_2)} - p_{c,C(M_1)+}\right)^2$$
$$\text{, for } C(M_1) = C(M_2) \text{ and } C(M_1), C(M_2) = 1, \cdots, C,$$

$\hat{V}_{o,1}, \hat{V}_{o,2}$ and $\hat{V}_{o,3}$ for $c = 1,2,3$ is defined as below:

$$\hat{V}_{o,1} =$$
$$\begin{bmatrix} p_{1,1+} + p_{1,+1} - 2p_{1,11} - (p_{1,+1} - p_{1,1+})^2 & -(p_{1,12} + p_{1,21}) - (p_{1,+1} + p_{1,1+})(p_{1,+2} + p_{1,2+}) \\ -(p_{1,21} + p_{1,12}) - (p_{1,+2} + p_{1,2+})(p_{1,+1} + p_{1,1+}) & p_{1,2+} + p_{1,+2} - 2p_{1,22} - (p_{1,+2} - p_{1,2+})^2 \end{bmatrix},$$

$$\hat{V}_{o,2} = [p_{2,2+} + p_{2,+2} - 2p_{2,22} - (p_{2,+2} - p_{2,2+})^2],$$

$\hat{V}_{o,3} =$

$$\begin{bmatrix} p_{3,3+} + p_{3,+3} - 2p_{3,33} - (p_{3,+3} - p_{3,3+})^2 & -(p_{3,32} + p_{3,23}) - (p_{3,+3} + p_{3,3+})(p_{3,+2} + p_{3,2+}) \\ -(p_{3,23} + p_{3,32}) - (p_{3,+2} + p_{3,2+})(p_{3,+3} + p_{3,3+}) & p_{3,2+} + p_{3,+2} - 2p_{3,22} - (p_{3,+2} - p_{3,2+})^2 \end{bmatrix}.$$

Z_{NRI_o} is the test statistic for testing the null hypothesis which is $NRI_o = 0$, and it is calculated as

$$Z_{NRI_o} = \frac{\hat{\theta}_o}{\sqrt{\hat{\sigma}_o^2}},$$

and compared to a standard normal distribution.

IV. Simulation

4.1. Simulation setting

To examine the performance of our proposed method of NRI, we consider fourteen different scenarios, which were constructed as described in Li et al.'s (2013) simulation settings. In scenarios 1 to 7, data were generated from a multinomial logistic structure, while data in scenarios 8 to 14 were generated from an ordinal logistic structure.

In scenario 1, a three-category response with the following multinomial logistic structures is used:

$$\log\frac{p_2}{p_1} = -0.5 + 1.5X_1 + 3X_2,$$

$$\log\frac{p_3}{p_1} = -1 + 2X_1 + 4X_2,$$

$$p_j = P(Y = j) \text{ for } j = 1,2,3.$$

Generate $(X_1,\ X_2)$ from a multivariate normal distribution with mean (1, 1) and covariate matrix $\sum = (\sigma_{i,j})_{1\le i,j\le 2}$. We let $\sigma_{11} = \sigma_{22} = 1$ and $\sigma_{12} = \sigma_{21} = 0$, and let $M_1 = \{X_1\}, M_2 = \{X_1, X_2\}$ in scenario 1. In scenario 2, we consider $\sigma_{11} = \sigma_{22} = 1$, and $\sigma_{12} = \sigma_{21} = 0.2$, and we divided the generated X_2 into a binary variable $X_2{}'$ based on 0 arbitrarily, and then consider $M_1 = \{X_1\}, M_2 = \{X_1, X_2'\}$ in scenario 3.

In scenario 4, a three-category response with the following multinomial logistic structures is used:

$$\log\frac{p_2}{p_1} = -2 + 0.25X_1 + 0.5X_2 + 0.75X_3 + 1X_4 + 1.25X_5,$$

$$\log\frac{p_3}{p_1} = -4 + 2.5X_1 + 2.25X_2 + 2X_3 + 1.75X_4 + 1.5X_5,$$

$$p_j = P(Y = j) \text{ for } j = 1,2,3.$$

Generate $(X_1,\ X_2,\ X_3,\ X_4,\ X_5)$ from a multivariate normal distribution with mean (1, 1, 1, 1, 1) and covariate matrix $\sum = (\sigma_{i,j})_{1\le i,j\le 5}$. We set $\sum = \text{diag}\{1,1,1,1,1\}$ and let $M_1 = \{X_1\}$, $M_2 = \{X_1, X_2, X_3, X_4, X_5\}$. In scenario 5, we set $\sigma_{11} = \sigma_{22} = \sigma_{33} = \sigma_{44} = \sigma_{55} = 1$, $\sigma_{12} = \sigma_{13} = \sigma_{14} = \sigma_{15} = \sigma_{23} = \sigma_{24} = \sigma_{25} = \sigma_{34} = \sigma_{35} = \sigma_{45} = 0.1$.

In scenario 6, a five-category response with the following multinomial logistic structures is used:

$$\log\frac{p_2}{p_1} = -4 + 3X_1 + 3.5X_2 + 4X_3,$$

$$\log\frac{p_3}{p_1} = -3 + 2.5X_1 + 3X_2 + 3.5X_3,$$

$$\log\frac{p_4}{p_1} = -2 + 2X_1 + 2.5X_2 + 3X_3,$$

$$\log\frac{p_5}{p_1} = -1 + 1.5X_1 + 2X_2 + 2.5X_3,$$

$$p_j = P(Y = j) \text{ for } j = 1,2,3,4,5.$$

Generate $(X_1,\ X_2,\ X_3)$ from a multivariate normal distribution with mean (1, 1, 1) and covariate matrix $\sum = (\sigma_{i,j})_{1\le i,j\le 5}$. We set $\sigma_{11} = \sigma_{22} = \sigma_{33} = 1$, $\sigma_{12} = \sigma_{13} = \sigma_{23} = 0$ and let $M_1 = \{X_1\}$, $M_2 = \{X_1, X_2, X_3\}$. In scenario 7, we set $\sigma_{11} = \sigma_{22} = \sigma_{33} = 1$, $\sigma_{12} = \sigma_{13} = \sigma_{23} = 0.1$.

In scenario 8, a three-category response with the following ordinal logistic structures is used:

$$\log\frac{p_2+p_3}{p_1}=-2+0.5X_1+2X_2,$$

$$\log\frac{p_3}{p_1+p_2}=-1+0.5X_1+2X_2,$$

$$p_j=P(Y=j)\text{ for } j=1,2,3.$$

Generate $(X_1,\ X_2)$ from a multivariate normal distribution with mean (1, 1) and covariate matrix $\sum=(\sigma_{i,j})_{1\le i,j\le 2}$. We let $\sigma_{11}=\sigma_{22}=1$ and $\sigma_{12}=\sigma_{21}=0$ and let $M_1=\{X_1\}, M_2=\{X_1,X_2\}$. $\sigma_{11}=\sigma_{22}=1$, $\sigma_{12}=\sigma_{21}=0.2$ and $M_1=\{X_1\}, M_2=\{X_1,X_2\}$ was set in scenario 9. We divided the generated X_2 into a binary variable $X_2{}'$ based on 0 arbitrarily, and then consider $M_1=\{X_1\}, M_2=\{X_1,X_2'\}$ in scenario 10.

In scenario 11, a three-category response with the following ordinal logistic structures is used:

$$\log\frac{p_2+p_3}{p_1}=-2+0.25X_1+0.5X_2+0.75X_3+1X_4+1.25X_5,$$

$$\log\frac{p_3}{p_1+p_2}=-1+0.25\mathrm{X}_1+0.5\mathrm{X}_2+0.75\mathrm{X}_3+1\mathrm{X}_4+1.25\mathrm{X}_5,$$

$$\mathrm{p}_j=P(Y=j)\text{ for } j=1,2,3.$$

Generate $(X_1,\ X_2,\ X_3,\ X_4,\ X_5)$ from a multivariate normal distribution with mean (1, 1, 1, 1, 1) and covariate matrix $\sum=(\sigma_{i,j})_{1\le i,j\le 5}$. We then set $\sum=\mathrm{diag}\{1,1,1,1,1\}$ and let $M_1 = \{X_1\}, M_2 = \{X_1, X_2, X_3, X_4, X_5\}$. We set $\sigma_{11} = \sigma_{22} = \sigma_{33} = \sigma_{44} = \sigma_{55} = 1$, $\sigma_{12} = \sigma_{13} = \sigma_{14} = \sigma_{15} = \sigma_{23} = \sigma_{24} = \sigma_{25} = \sigma_{34} = \sigma_{35} = \sigma_{45} = 0.1$ in scenario 12.

In scenario 13, a five-category response with the following ordinal logistic structures is used:

$$\log\frac{p_2 + p_3 + p_4 + p_5}{p_1} = -4 + 0.5X_1 + 1.5X_2 + 2.5X_3,$$

$$\log\frac{p_3 + p_4 + p_5}{p_1 + p_2} = -3 + 0.5X_1 + 1.5X_2 + 2.5X_3,$$

$$\log\frac{p_4 + p_5}{p_1 + p_2 + p_3} = -2 + 0.5X_1 + 1.5X_2 + 2.5X_3,$$

$$\log\frac{p_5}{p_1 + p_2 + p_3 + p_4} = -1 + 0.5X_1 + 1.5X_2 + 2.5X_3,$$

$$\mathrm{p}_j = P(Y = j) \text{ for } j = 1,2,3,4,5.$$

Generate $(X_1,\ X_2,\ X_3)$ from a multivariate normal distribution with mean (1, 1, 1) and covariate matrix $\sum = (\sigma_{i,j})_{1\le i,j\le 5}$. We set $\sigma_{11} = \sigma_{22} = \sigma_{33} = 1$, $\sigma_{12} = \sigma_{13} = \sigma_{23} = 0$ and let $M_1 = \{X_1\}, M_2 = \{X_1, X_2, X_3\}$. In scenario 14, we set $\sigma_{11} = \sigma_{22} = \sigma_{33} = 1$, $\sigma_{12} = \sigma_{13} = \sigma_{23} = 0.1$.

In scenarios 1 to 5 and scenarios 8 to 12, Y is generated from a uniform distribution. When Y is smaller than p_1, between p_1 and p_2, or greater than or equal to p_2, then it is defined to group 1, group 2 or group 3, respectively. The three groups are defined as outcomes, and then 1 vs. 2/3 and 1/2 vs.3 are additionally classified.

In scenarios 6 and 7 and scenarios 13 and 14, Y is generated from a uniform distribution. When Y is smaller than p_1, between p_1 and p_2, between p_2 and p_3, between p_3 and p_4, or greater than or equal to p_4, then it is defined as group 1, group 2, group 3, group 4 or group 5, respectively. The five groups are defined as outcomes, and then 1/2 vs. 3/4/5 and 1/2/3 vs. 4/5 are additionally classified.

Next, we then consider the results obtained from Monte Carlo simulations conducted with 100000 simulations as true values for five methods. The true values consisted of NRI_{o,w_1}, NRI_{o,w_2}, and NRI_{o,w_3} obtained by the newly proposed method; RI_m and NRI_m obtained by Li's method; NRI_b obtained by Pencina's method; ΔVUS obtained by Nakas's method; and ΔAUC obtained by Delong's method.

The proposed method was performed with 1000 simulations to obtain $\text{Avg.}\hat{\theta}$, relative bias, and $\text{Avg.}SE_{\hat{\theta}}$ and $SD_{\hat{\theta}}$, and coverage rate was confirmed. θ represents the true value using Monte Carlo, $\text{Avg.}\hat{\theta}$ refer to the mean of estimated θ over 1000 simulations, and relative bias equals $\left(\frac{\theta-\text{Avg.}\hat{\theta}}{\theta}\right)$. $\text{Avg.}SE_{\hat{\theta}}$ is the mean of the estimated standard error of $\text{Avg.}\hat{\theta}$. The proposed method was computed, according to the Stuart-Maxwell test and Bhapkar's test, while the other methods were computed using one formula. $SD_{\hat{\theta}}$ is the standard deviation of the estimated θ over 1000 simulations. CR equals (the number of times the simulations of the true NRI fell into the interval $[\text{Avg.}\hat{\theta} - 1.96\text{Avg.}SE_{\hat{\theta}}, \text{Avg.}\hat{\theta} + 1.96\text{Avg.}SE_{\hat{\theta}}]$)/1000.

4.2. Simulation results

The simulation results for the proposed method demonstrated NRI with w_2 to be the largest and that with w_1 to be the smallest among NRIs considering three weights, as shown in Tables 6 to 19.

Relative bias was greater when the number of additional variables increased to more than two (Tables 9 to 12), compared to adding one variable (Tables 6 to 8), in multinomial structure settings; relative bias was remained stable in ordinal structure settings. Relative bias in Li's method (Tables 18 and Table 19) was greater than that in the proposed method when based on an ordinal logistic structure.

The standard error of the newly proposed method estimated using Stuart-Maxwell's test was better than the SE estimated using Bhapkar's test in regards to coverage rate (Tables 6 to 19).

When continuous scale variables (Table 6, Table 13) were categorized arbitrarily (Table8, Table 15), the coverage rate reduced tremendously. The coverage rate was similar n=150 and n=300. When a correlation between a newly added factor and existing known factors was present, the coverage rate in the proposed method was reduced. The coverage rate of the proposed method tended to be similar or slightly higher than the RIs and NRIs obtained by Li's method (Tables 6 to 14 and Tables 16 to 19). Pencina's method generally showed high coverage rates, but although a relatively low coverage rate was noted upon categorizing the outcome variables in a different way (Table 11, Table 12).

ΔVUS could not be obtained since Nakas's method takes a long time to compute when estimating SE or when numerous outcome categories are present (Tables 6 to 19).

Although $\Delta\mathrm{AUC}$ in Delong's method is widely used, the coverage rate thereof was the lowest, compared to the other measures, and the results for AUC become incoherent upon categorizing outcome variables in a different way (Table 6 to 19).

Scatter plots of NRI and $\Delta\mathrm{VUS}$ in scenario 1 and scenario 8 are shown in Figure 1 and Figure 2, and the degree of correlation is also presented. The correlation between NRI which considered w_1, w_2, and w_3 and $\Delta\mathrm{VUS}$, showed strong correlation of almost 0.9; the correlation in w_3 was especially high. $\Delta\mathrm{VUS}$ often cannot be obtained due to the complexity of its calculation. However, NRI for ordinal outcomes could replace $\Delta\mathrm{VUS}$ for confirming predictive ability with adding new factors since the correlation between NRI and $\Delta\mathrm{VUS}$ show strong correlation.

Table 6. Simulation results based on a multinomial logistic structure – scenario 1

	Outcome†	θ	Avg. $\hat{\theta}$	Relative bias	Avg. $SE_{\hat{\theta}}$	$SD_{\hat{\theta}}$	CR
n=150	Proposed method						
	1 vs.2 vs. 3 (NRI_{o,w_1})	0.778031	0.751570	0.034010	0.207038/0.104932	0.133076	0.988/0.863
	1 vs.2 vs. 3 (NRI_{o,w_2})	1.150593	1.125000	0.022243	0.318483/0.172917	0.206361	0.990/0.887
	1 vs.2 vs. 3 (NRI_{o,w_3})	0.817108	0.813779	0.004074	-	0.164261	-
	Li's method						
	1 vs. 2 vs. 3 (RI_m)	0.248375	0.248950	-0.002316	0.067917	0.051043	0.985
	1 vs. 2 vs. 3 (NRI_m)	0.267628	0.272967	-0.019950	0.066289	0.053569	0.981
	Pencina's method						
	1 vs. 2/3 (NRI_b)	0.600189	0.586011	0.023623	0.182386	0.136612	0.977
	1/2 vs. 3 (NRI_b)	0.349206	0.338910	0.029484	0.104382	0.102321	0.938
	Nakas's method						
	1 vs.2 vs. 3 (ΔVUS)	-	0.283621	-	-	0.064888	-
	Delong's method						
	1 vs. 2/3 (ΔAUC)	0.300095	0.293006	0.034010	0.061315	0.068306	0.906
	1/2 vs. 3 (ΔAUC)	0.174603	0.169455	0.022243	0.042299	0.051161	0.892
n=300	Proposed method						
	1 vs.2 vs. 3 (NRI_{o,w_1})	0.778031	0.761763	0.020909	0.147510/0.074146	0.090488	0.990/0.893
	1 vs.2 vs. 3 (NRI_{o,w_2})	1.150593	1.132875	0.015399	0.226494/0.121944	0.139968	0.993/0.903
	1 vs.2 vs. 3 (NRI_{o,w_3})	0.817108	0.810812	0.007706	-	0.114836	-
	Li's method						
	1 vs. 2 vs. 3 (RI_m)	0.248375	0.247408	0.000389	0.047946	0.034692	0.984
	1 vs. 2 vs. 3 (NRI_m)	0.267628	0.269849	-0.008298	0.046877	0.036643	0.981
	Pencina's method						
	1 vs. 2/3 (NRI_b)	0.600189	0.589569	0.017695	0.130938	0.092633	0.988
	1/2 vs. 3 (NRI_b)	0.349206	0.344124	0.014552	0.072890	0.072806	0.944
	Nakas's method						
	1 vs.2 vs. 3 (ΔVUS)	-	0.281995	-	-	0.064888	-
	Delong's method						
	1 vs. 2/3 (ΔAUC)	0.300095	0.294784	0.017695	0.043121	0.046317	0.923
	1/2 vs. 3 (ΔAUC)	0.174603	0.172062	0.014552	0.030002	0.036403	0.891

† Prevalence of outcome - 1: 11.44%, 2: 23.48%, 3: 65.08%

Table 7. Simulation results based on a multinomial logistic structure – scenario 2

	Outcome†	θ	Avg. $\hat{\theta}$	Relative bias	Avg. $SE_{\hat{\theta}}$	$SD_{\hat{\theta}}$	CR
n=150	Proposed method						
	1 vs.2 vs. 3 (NRI_{o,w_1})	0.667513	0.656734	0.016148	0.182204/0.111931	0.141106	0.963/0.857
	1 vs.2 vs. 3 (NRI_{o,w_2})	0.989209	0.988636	0.000579	0.279534/0.178798	0.216866	0.969/0.870
	1 vs.2 vs. 3 (NRI_{o,w_3})	0.702767	0.720088	-0.024647	-	0.174630	-
	Li's method						
	1 vs. 2 vs. 3 (RI_m)	0.214464	0.221268	-0.031726	0.061453	0.052884	0.969
	1 vs. 2 vs. 3 (NRI_m)	0.237187	0.248887	-0.049328	0.059059	0.054492	0.959
	Pencina's method						
	1 vs. 2/3 (NRI_b)	0.568851	0.555978	0.022630	0.175537	0.130348	0.970
	1/2 vs. 3 (NRI_b)	0.278169	0.280024	-0.006669	0.095775	0.104896	0.928
	Nakas's method						
	1 vs.2 vs. 3 (ΔVUS)	-	0.256064	-	-	0.065411	-
	Delong's method						
	1 vs. 2/3 (ΔAUC)	0.284426	0.277989	0.022630	0.060230	0.065174	0.919
	1/2 vs. 3 (ΔAUC)	0.139085	0.140012	-0.006668	0.042680	0.052448	0.889
n=300	Proposed method						
	1 vs.2 vs. 3 (NRI_{o,w_1})	0.667513	0.667418	0.000142	0.130118/0.080446	0.101383	0.982/0.859
	1 vs.2 vs. 3 (NRI_{o,w_2})	0.989209	0.997785	-0.008670	0.199020/0.127593	0.155814	0.982/0.870
	1 vs.2 vs. 3 (NRI_{o,w_3})	0.702767	0.718050	-0.021747	-	0.124767	-
	Li's method						
	1 vs. 2 vs. 3 (RI_m)	0.214464	0.220244	-0.026951	0.043294	0.037864	0.970
	1 vs. 2 vs. 3 (NRI_m)	0.237187	0.245786	-0.036254	0.041710	0.038949	0.958
	Pencina's method						
	1 vs. 2/3 (NRI_b)	0.568851	0.564427	0.007777	0.125935	0.090650	0.982
	1/2 vs. 3 (NRI_b)	0.278169	0.281169	-0.010785	0.067592	0.071473	0.929
	Nakas's method						
	1 vs.2 vs. 3 (ΔVUS)	-	0.254991	-	-	0.047078	-
	Delong's method						
	1 vs. 2/3 (ΔAUC)	0.284426	0.282213	0.007779	0.042355	0.045325	0.918
	1/2 vs. 3 (ΔAUC)	0.139085	0.140584	-0.010780	0.030367	0.035736	0.894

† Prevalence of outcome - 1: 1: 12.92%, 2: 22.68%, 3: 64.39%

Table 8. Simulation results based on a multinomial logistic structure – scenario 3

	Outcome†	θ	Avg. $\hat{\theta}$	Relative bias	Avg. $SE_{\hat{\theta}}$	$SD_{\hat{\theta}}$	CR
n=150	Proposed method						
	1 vs.2 vs. 3 (NRI_{o,w_1})	0.347721	0.353484	-0.016570	0.137514/0.104652	0.157102	0.886/0.786
	1 vs.2 vs. 3 (NRI_{o,w_2})	0.521559	0.511606	0.019083	0.221078/0.170834	0.229290	0.906/0.834
	1 vs.2 vs. 3 (NRI_{o,w_3})	0.347677	0.386853	-0.112680	-	0.183121	-
	Li's method						
	1 vs. 2 vs. 3 (RI_m)	0.115892	0.105415	0.090403	0.045547	0.051641	0.860
	1 vs. 2 vs. 3 (NRI_m)	0.131330	0.130419	0.006937	0.043494	0.056876	0.834
	Pencina's method						
	1 vs. 2/3 (NRI_b)	0.166206	0.159513	0.040269	0.158313	0.129162	0.839
	1/2 vs. 3 (NRI_b)	0.274887	0.258262	0.060479	0.104143	0.116660	0.898
	Nakas's method						
	1 vs.2 vs. 3 (ΔVUS)	-	0.119896	-	-	0.060003	-
	Delong's method						
	1 vs. 2/3 (ΔAUC)	0.083103	0.079757	0.040263	0.040741	0.064581	0.699
	1/2 vs. 3 (ΔAUC)	0.137443	0.129131	0.060476	0.043311	0.058330	0.842
n=300	Proposed method						
	1 vs.2 vs. 3 (NRI_{o,w_1})	0.347721	0.350418	-0.007760	0.101780/0.079754	0.112175	0.904/0.817
	1 vs.2 vs. 3 (NRI_{o,w_2})	0.521559	0.511329	0.019614	0.159588/0.126710	0.162937	0.913/0.861
	1 vs.2 vs. 3 (NRI_{o,w_3})	0.347677	0.371470	-0.068430	-	0.128338	-
	Li's method						
	1 vs. 2 vs. 3 (RI_m)	0.115892	0.107261	0.074475	0.033462	0.036289	0.883
	1 vs. 2 vs. 3 (NRI_m)	0.131330	0.128278	0.023239	0.032323	0.039691	0.865
	Pencina's method						
	1 vs. 2/3 (NRI_b)	0.166206	0.150477	0.094636	0.119040	0.096886	0.913
	1/2 vs. 3 (NRI_b)	0.274887	0.262013	0.046834	0.072983	0.082984	0.901
	Nakas's method						
	1 vs.2 vs. 3 (ΔVUS)	-	0.115262	-	-	0.042920	-
	Delong's method						
	1 vs. 2/3 (ΔAUC)	0.083103	0.075238	0.094642	0.030073	0.048443	0.737
	1/2 vs. 3 (ΔAUC)	0.137443	0.131006	0.046834	0.031049	0.041492	0.838

† Prevalence of outcome - 1: 11.44%, 2: 23.48%, 3: 65.08%

Table 9. Simulation results based on a multinomial logistic structure – scenario 4

	Outcome†	θ	Avg. $\hat{\theta}$	Relative bias	Avg. $SE_{\hat{\theta}}$	$SD_{\hat{\theta}}$	CR
n=150	Proposed method						
	1 vs.2 vs. 3 (NRI_{o,w_1})	0.716378	0.799242	-0.115670	0.249139/0.142121	0.168361	0.996/0.841
	1 vs.2 vs. 3 (NRI_{o,w_2})	1.114961	1.261353	-0.131298	0.358313/0.193298	0.365859	0.982/0.775
	1 vs.2 vs. 3 (NRI_{o,w_3})	0.840981	0.965629	-0.148217	-	0.227600	-
	Li's method						
	1 vs. 2 vs. 3 (RI_m)	0.265680	0.308075	-0.159571	0.094034	0.076913	0.981
	1 vs. 2 vs. 3 (NRI_m)	0.282143	0.324201	-0.149065	0.092254	0.073861	0.984
	Pencina's method						
	1 vs. 2/3 (NRI_b)	0.474282	0.554840	-0.169854	0.212678	0.173967	0.943
	1/2 vs. 3 (NRI_b)	0.466949	0.495762	-0.061706	0.152270	0.133740	0.964
	Nakas's method						
	1 vs.2 vs. 3 (ΔVUS)	-	0.350583	-	-	0.108587	-
	Delong's method						
	1 vs. 2/3 (ΔAUC)	0.237141	0.277420	-0.169854	0.071540	0.086984	0.820
	1/2 vs. 3 (ΔAUC)	0.233474	0.247881	-0.061706	0.055629	0.066870	0.861
n=300	Proposed method						
	1 vs.2 vs. 3 (NRI_{o,w_1})	0.716378	0.804435	-0.122919	0.249715	0.173605	0.994/0.825
	1 vs.2 vs. 3 (NRI_{o,w_2})	1.114961	1.268728	-0.137912	0.358357/0.189737	0.287548	0.980/0.754
	1 vs.2 vs. 3 (NRI_{o,w_3})	0.840981	0.969380	-0.152678	-	0.231304	-
	Li's method						
	1 vs. 2 vs. 3 (RI_m)	0.265680	0.309528	-0.165040	0.095741	0.078961	0.986
	1 vs. 2 vs. 3 (NRI_m)	0.282143	0.324674	-0.150742	0.094284	0.075876	0.987
	Pencina's method						
	1 vs. 2/3 (NRI_b)	0.474282	0.561549	-0.183999	0.210019	0.178433	0.925
	1/2 vs. 3 (NRI_b)	0.466949	0.495928	-0.062061	0.152458	0.135455	0.969
	Nakas's method						
	1 vs.2 vs. 3 (ΔVUS)	-	0.351259	-	-	0.112706	-
	Delong's method						
	1 vs. 2/3 (ΔAUC)	0.237141	0.280774	-0.183997	0.070603	0.089217	0.799
	1/2 vs. 3 (ΔAUC)	0.233474	0.247964	-0.062061	0.055738	0.067727	0.857

† Prevalence of outcome - 1: 8.11%, 2: 8.25%, 3: 83.86%

Table 10. Simulation results based on a multinomial logistic structure – scenario 5

	Outcome†	θ	Avg. $\hat{\theta}$	Relative bias	Avg. $SE_{\hat{\theta}}$	$SD_{\hat{\theta}}$	CR
n=150	Proposed method						
	1 vs.2 vs. 3 (NRI_{o,w_1})	0.751235	0.792412	-0.054810	0.218569/0.129977	0.158942	0.992/0.872
	1 vs.2 vs. 3 (NRI_{o,w_2})	1.173190	1.253505	-0.068460	0.297660/0.159128	0.262395	0.962/0.759
	1 vs.2 vs. 3 (NRI_{o,w_3})	0.872425	0.952113	-0.091341	-	0.214896	-
	Li's method						
	1 vs. 2 vs. 3 (RI_m)	0.281303	0.307395	-0.092756	0.085231	0.072237	0.981
	1 vs. 2 vs. 3 (NRI_m)	0.299597	0.326507	-0.089822	0.082968	0.069032	0.982
	Pencina's method						
	1 vs. 2/3 (NRI_b)	0.550944	0.594995	-0.079955	0.186240	0.145784	0.961
	1/2 vs. 3 (NRI_b)	0.458357	0.477940	-0.042724	0.138615	0.128291	0.965
	Nakas's method						
	1 vs.2 vs. 3 (ΔVUS)	-	0.361373	-	-	0.101836	-
	Delong's method						
	1 vs. 2/3 (ΔAUC)	0.275472	0.297497	-0.079953	0.061625	0.072892	0.850
	1/2 vs. 3 (ΔAUC)	0.229179	0.238970	-0.042723	0.052378	0.064146	0.871
n=300	Proposed method						
	1 vs.2 vs. 3 (NRI_{o,w_1})	0.751235	0.792332	-0.054706	0.218494/0.129727	0.160823	0.991/0.852
	1 vs.2 vs. 3 (NRI_{o,w_2})	1.173189	1.252329	-0.067457	0.297449/0.156698	0.266387	0.961/0.741
	1 vs.2 vs. 3 (NRI_{o,w_3})	0.872425	0.950268	-0.089226	-	0.220354	-
	Li's method						
	1 vs. 2 vs. 3 (RI_m)	0.281303	0.306664	-0.090157	0.086963	0.073618	0.983
	1 vs. 2 vs. 3 (NRI_m)	0.299597	0.325432	-0.086234	0.084973	0.069927	0.983
	Pencina's method						
	1 vs. 2/3 (NRI_b)	0.550944	0.595165	-0.080263	0.184466	0.145109	0.960
	1/2 vs. 3 (NRI_b)	0.458357	0.481549	-0.050597	0.139445	0.129039	0.963
	Nakas's method						
	1 vs.2 vs. 3 (ΔVUS)	-	0.360432	-	-	0.102340	-
	Delong's method						
	1 vs. 2/3 (ΔAUC)	0.275472	0.297583	-0.080265	0.061495	0.072555	0.859
	1/2 vs. 3 (ΔAUC)	0.229179	0.240775	-0.050599	0.052599	0.064519	0.869

† Prevalence of outcome - 1: 11.07%, 2: 8.00%, 3: 80.93%

Table 11. Simulation results based on a multinomial logistic structure – scenario 6

	Outcome†	θ	Avg. $\hat{\theta}$	Relative bias	Avg. $SE_{\hat{\theta}}$	$SD_{\hat{\theta}}$	CR
n=150	Proposed method						
	1 vs.2 vs. 3 (NRI_{o,w_1})	0.386563	0.487147	-0.260201	0.204993/0.177160	0.204830	0.948/0.849
	1 vs.2 vs. 3 (NRI_{o,w_2})	1.135120	1.290131	-0.136559	0.403257/0.312708	0.345207	0.976/0.861
	1 vs.2 vs. 3 (NRI_{o,w_3})	0.843737	1.024970	-0.214798	-	0.287005	-
	Li's method						
	1 vs. 2 vs. 3 (RI_m)	0.199615	0.214129	-0.072710	0.058162	0.045037	0.986
	1 vs. 2 vs. 3 (NRI_m)	0.222190	0.246182	-0.107980	0.055632	0.046265	0.974
	Pencina's method						
	1 vs. 2/3 (NRI_b)	0.144563	0.138556	0.041553	0.082810	0.095863	0.881
	1/2 vs. 3 (NRI_b)	0.023322	0.070600	-2.027184	0.064024	0.095536	0.783
	Nakas's method						
	1 vs.2 vs. 3 (ΔVUS)	-	-	-	-	-	-
	Delong's method						
	1 vs. 2/3 (ΔAUC)	0.072281	0.069278	0.041546	0.040110	0.047931	0.899
	1/2 vs. 3 (ΔAUC)	0.011661	0.035300	-2.027185	0.030344	0.047768	0.767
n=300	Proposed method						
	1 vs.2 vs. 3 (NRI_{o,w_1})	0.386563	0.455441	-0.178181	0.144704/0.130836	0.149682	0.922/0.867
	1 vs.2 vs. 3 (NRI_{o,w_2})	1.135120	1.230617	-0.084129	0.281078/0.227535	0.248026	0.974/0.902
	1 vs.2 vs. 3 (NRI_{o,w_3})	0.843740	0.971478	-0.151395	-	0.207851	-
	Li's method						
	1 vs. 2 vs. 3 (RI_m)	0.199615	0.206712	-0.035553	0.040374	0.032106	0.986
	1 vs. 2 vs. 3 (NRI_m)	0.222190	0.236298	-0.006350	0.038643	0.032116	0.979
	Pencina's method						
	1 vs. 2/3 (NRI_b)	0.144563	0.136484	0.055886	0.058880	0.072735	0.865
	1/2 vs. 3 (NRI_b)	0.023322	0.050648	-1.171683	0.044092	0.072757	0.763
	Nakas's method						
	1 vs.2 vs. 3 (ΔVUS)	-	-	-	-	-	-
	Delong's method						
	1 vs. 2/3 (ΔAUC)	0.072281	0.068242	0.055879	0.028909	0.036368	0.856
	1/2 vs. 3 (ΔAUC)	0.011661	0.025324	-1.171683	0.021024	0.036378	0.766

† Prevalence of outcome -1: 7.34%, 2: 44.41%, 3: 20.41%, 4: 14.01%,

Table 12. Simulation results based on a multinomial logistic structure – scenario 7

	Outcome†	θ	Avg. $\hat{\theta}$	Relative bias	Avg. $SE_{\hat{\theta}}$	$SD_{\hat{\theta}}$	CR
n=150	Proposed method						
	1 vs.2 vs. 3 (NRI_{o,w_1})	0.408201	0.507425	-0.243076	0.195952/0.168688	0.197002	0.937/0.842
	1 vs.2 vs. 3 (NRI_{o,w_2})	1.147460	1.309075	-0.140846	0.386156/0.302477	0.341762	0.966/0.863
	1 vs.2 vs. 3 (NRI_{o,w_3})	0.864296	1.043127	-0.206909	-	0.278180	-
	Li's method						
	1 vs. 2 vs. 3 (RI_m)	0.197136	0.213782	-0.084439	0.051410	0.045885	0.976
	1 vs. 2 vs. 3 (NRI_m)	0.220553	0.246171	-0.116154	0.052365	0.046238	0.953
	Pencina's method						
	1 vs. 2/3 (NRI_b)	0.126962	0.124481	0.019541	0.080360	0.096336	0.865
	1/2 vs. 3 (NRI_b)	-0.006761	0.047864	8.079426	0.059038	0.087369	0.969
	Nakas's method						
	1 vs.2 vs. 3 (ΔVUS)	-	-	-	-	-	-
	Delong's method						
	1 vs. 2/3 (ΔAUC)	0.063481	0.062240	0.019549	0.039075	0.043860	0.889
	1/2 vs. 3 (ΔAUC)	-0.003380	0.023932	8.080473	0.028361	0.043684	0.998
n=300	Proposed method						
	1 vs.2 vs. 3 (NRI_{o,w_1})	0.408201	0.479278	-0.174123	0.138590/0.123620	0.141615	0.925/0.868
	1 vs.2 vs. 3 (NRI_{o,w_2})	1.147460	1.249220	-0.088683	0.269535/0.218462	0.241961	0.970/0.897
	1 vs.2 vs. 3 (NRI_{o,w_3})	0.864296	0.988757	-0.144002	-	0.200200	-
	Li's method						
	1 vs. 2 vs. 3 (RI_m)	0.197136	0.205322	-0.041525	0.038282	0.032113	0.975
	1 vs. 2 vs. 3 (NRI_m)	0.220553	0.235095	-0.065934	0.036408	0.032251	0.959
	Pencina's method						
	1 vs. 2/3 (NRI_b)	0.126962	0.118622	0.065689	0.057204	0.074930	0.838
	1/2 vs. 3 (NRI_b)	-0.006761	0.025294	4.741163	0.040042	0.063774	0.941
	Nakas's method						
	1 vs.2 vs. 3 (ΔVUS)	-	-	-	-	-	-
	Delong's method						
	1 vs. 2/3 (ΔAUC)	0.063481	0.059311	0.065689	0.028084	0.037465	0.839
	1/2 vs. 3 (ΔAUC)	-0.003380	0.012647	4.741716	0.019297	0.031887	0.996

† Prevalence of outcome -1: 8.41%, 2: 44.41%, 3: 19.86%, 4: 13.59%, 5: 13.73%

Table 13. Simulation results based on an ordinal logistic structure – scenario 8

	Outcome†	θ	Avg. $\hat{\theta}$	Relative bias	Avg. $SE_{\hat{\theta}}$	$SD_{\hat{\theta}}$	CR
n=150	Proposed method						
	1 vs.2 vs. 3 (NRI_{o,w_1})	0.665206	0.649969	0.022906	0.131615/0.096567	0.126679	0.950/0.863
	1 vs.2 vs. 3 (NRI_{o,w_2})	0.997755	0.985395	0.012388	0.182669/0.142348	0.199600	0.929/0.834
	1 vs.2 vs. 3 (NRI_{o,w_3})	0.777011	0.772229	0.006154	-	0.166350	-
	Li's method						
	1 vs. 2 vs. 3 (RI_m)	0.221700	0.223610	-0.008620	0.049564	0.051714	0.949
	1 vs. 2 vs. 3 (NRI_m)	0.282168	0.284737	-0.009100	0.043474	0.051660	0.900
	Pencina's method						
	1 vs. 2/3 (NRI_b)	0.354382	0.374000	-0.055360	0.097880	0.093503	0.957
	1/2 vs. 3 (NRI_b)	0.453455	0.448159	0.011679	0.122933	0.101325	0.964
	Nakas's method						
	1 vs.2 vs. 3 (ΔVUS)	-	0.262367	-	-	0.058825	-
	Delong's method						
	1 vs. 2/3 (ΔAUC)	0.177191	0.187000	-0.055360	0.045887	0.046752	0.934
	1/2 vs. 3 (ΔAUC)	0.226728	0.224080	0.011677	0.045457	0.050663	0.902
n=300	Proposed method						
	1 vs.2 vs. 3 (NRI_{o,w_1})	0.665206	0.660905	0.006466	0.093619/0.069891	0.093808	0.957/0.847
	1 vs.2 vs. 3 (NRI_{o,w_2})	0.997755	0.998294	-0.000540	0.128678/0.102234	0.145696	0.917/0.809
	1 vs.2 vs. 3 (NRI_{o,w_3})	0.777011	0.780979	-0.005110	-	0.177982	-
	Li's method						
	1 vs. 2 vs. 3 (RI_m)	0.221700	0.224931	-0.014580	0.035270	0.036599	0.944
	1 vs. 2 vs. 3 (NRI_m)	0.282168	0.286464	-0.015220	0.030911	0.036324	0.900
	Pencina's method						
	1 vs. 2/3 (NRI_b)	0.354382	0.366405	-0.033930	0.068876	0.066255	0.956
	1/2 vs. 3 (NRI_b)	0.453455	0.452271	0.002611	0.087236	0.071174	0.978
	Nakas's method						
	1 vs.2 vs. 3 (ΔVUS)	-	0.266317	-	-	0.041092	-
	Delong's method						
	1 vs. 2/3 (ΔAUC)	0.177191	0.183203	-0.033930	0.032599	0.033127	0.946
	1/2 vs. 3 (ΔAUC)	0.226728	0.226136	0.002609	0.032072	0.035587	0.921

† Prevalence of outcome -1: 50.06%, 2: 25.91%, 3: 24.03%

Table 14. Simulation results based on an ordinal logistic structure – scenario 9

	Outcome†	θ	Avg. $\hat{\theta}$	Relative bias	Avg. $SE_{\hat{\theta}}$	$SD_{\hat{\theta}}$	CR
n=150	Proposed method						
	1 vs.2 vs. 3 (NRI_{o,w_1})	0.547682	0.537825	0.017997	0.118203/0.097694	0.123038	0.934/0.869
	1 vs.2 vs. 3 (NRI_{o,w_2})	0.827221	0.822923	0.005195	0.160823/0.141782	0.194964	0.894/0.836
	1 vs.2 vs. 3 (NRI_{o,w_3})	0.653541	0.655255	-0.002620	-	0.16226	-
	Li's method						
	1 vs. 2 vs. 3 (RI_m)	0.186359	0.190073	-0.019930	0.047470	0.051028	0.924
	1 vs. 2 vs. 3 (NRI_m)	0.246918	0.252178	-0.021300	0.040958	0.051358	0.869
	Pencina's method						
	1 vs. 2/3 (NRI_b)	0.294642	0.306868	-0.041500	0.090559	0.087752	0.962
	1/2 vs. 3 (NRI_b)	0.389090	0.392134	-0.007820	0.115408	0.106353	0.965
	Nakas's method						
	1 vs.2 vs. 3 (ΔVUS)	-	0.23353	-	-	0.058185	-
	Delong's method						
	1 vs. 2/3 (ΔAUC)	0.147321	0.153434	-0.041500	0.043359	0.043876	0.944
	1/2 vs. 3 (ΔAUC)	0.194545	0.196067	-0.007820	0.046290	0.053176	0.904
n=300	Proposed method						
	1 vs.2 vs. 3 (NRI_{o,w_1})	0.547682	0.545508	0.003969	0.08397/0.070406	0.088345	0.940/0.890
	1 vs.2 vs. 3 (NRI_{o,w_2})	0.827221	0.830970	-0.004530	0.112694/0.101176	0.138617	0.896/0.848
	1 vs.2 vs. 3 (NRI_{o,w_3})	0.653541	0.660926	-0.011300	-	0.113053	-
	Li's method						
	1 vs. 2 vs. 3 (RI_m)	0.186359	0.190311	-0.021210	0.033712	0.035389	0.934
	1 vs. 2 vs. 3 (NRI_m)	0.246918	0.252327	-0.021900	0.029102	0.036028	0.879
	Pencina's method						
	1 vs. 2/3 (NRI_b)	0.294642	0.304036	-0.031880	0.063832	0.062416	0.955
	1/2 vs. 3 (NRI_b)	0.389090	0.389779	-0.001770	0.081726	0.077398	0.965
	Nakas's method						
	1 vs.2 vs. 3 (ΔVUS)	-	0.237164	-	-	0.040745	-
	Delong's method						
	1 vs. 2/3 (ΔAUC)	0.147321	0.152018	-0.031880	0.030632	0.031208	0.942
	1/2 vs. 3 (ΔAUC)	0.194545	0.194889	-0.001770	0.032977	0.038699	0.890

† Prevalence of outcome -1: 50.07%, 2: 24.79%, 3: 25.14%

Table 15. Simulation results based on an ordinal logistic structure – scenario 10

	Outcome†	θ	Avg. $\hat{\theta}$	Relative bias	Avg. $SE_{\hat{\theta}}$	$SD_{\hat{\theta}}$	CR
n=150	Proposed method						
	1 vs.2 vs. 3 (NRI_{o,w_1})	0.516926	0.498881	0.034909	0.126618/0.098846	0.132837	0.934/0.852
	1 vs.2 vs. 3 (NRI_{o,w_2})	0.755495	0.734334	0.028009	0.184805/0.148801	0.200529	0.928/0.854
	1 vs.2 vs. 3 (NRI_{o,w_3})	0.578499	0.574222	0.007394	-	0.163530	-
	Li's method						
	1 vs. 2 vs. 3 (RI_m)	0.159032	0.156969	0.012972	0.046512	0.04802	0.941
	1 vs. 2 vs. 3 (NRI_m)	0.228439	0.231455	-0.013200	0.039345	0.047727	0.883
	Pencina's method						
	1 vs. 2/3 (NRI_b)	0.290124	0.316675	-0.09152	0.106918	0.103661	0.954
	1/2 vs. 3 (NRI_b)	0.296370	0.270140	0.088503	0.102107	0.119499	0.861
	Nakas's method						
	1 vs.2 vs. 3 (ΔVUS)	-	0.176896	-	-	0.052211	-
	Delong's method						
	1 vs. 2/3 (ΔAUC)	0.145062	0.158337	-0.091510	0.051283	0.051831	0.939
	1/2 vs. 3 (ΔAUC)	0.148185	0.135070	0.088502	0.043087	0.059750	0.821
n=300	Proposed method						
	1 vs.2 vs. 3 (NRI_{o,w_1})	0.516926	0.509831	0.013726	0.090859/0.072341	0.104378	0.913/0.819
	1 vs.2 vs. 3 (NRI_{o,w_2})	0.755495	0.748506	0.009251	0.131271/0.107705	0.157410	0.894/0.813
	1 vs.2 vs. 3 (NRI_{o,w_3})	0.578499	0.580363	-0.00322	-	0.128679	-
	Li's method						
	1 vs. 2 vs. 3 (RI_m)	0.159032	0.159115	-0.000520	0.032949	0.037302	0.909
	1 vs. 2 vs. 3 (NRI_m)	0.228439	0.23105	-0.011430	0.028052	0.037823	0.854
	Pencina's method						
	1 vs. 2/3 (NRI_b)	0.290124	0.306583	-0.056730	0.075785	0.072887	0.960
	1/2 vs. 3 (NRI_b)	0.296370	0.284512	0.040009	0.074696	0.083677	0.902
	Nakas's method						
	1 vs.2 vs. 3 (ΔVUS)	-	0.180298	-	-	0.041441	-
	Delong's method						
	1 vs. 2/3 (ΔAUC)	0.145062	0.153292	-0.056730	0.036687	0.036444	0.945
	1/2 vs. 3 (ΔAUC)	0.148185	0.142256	0.040009	0.031177	0.041839	0.835

† Prevalence of outcome -1: 50.06%, 2: 25.91%, 3: 24.03%

Table 16. Simulation results based on an ordinal logistic structure – scenario 11

	Outcome†	θ	Avg. $\hat{\theta}$	Relative bias	Avg. $SE_{\hat{\theta}}$	$SD_{\hat{\theta}}$	CR
n=150	Proposed method						
	1 vs.2 vs. 3 (NRI_{o,w_1})	0.704711	0.720428	-0.022302	0.165528/0.093183	0.122478	0.984/0.853
	1 vs.2 vs. 3 (NRI_{o,w_2})	1.034413	1.062478	-0.027131	0.244444/0.145220	0.196235	0.981/0.845
	1 vs.2 vs. 3 (NRI_{o,w_3})	0.829152	0.853121	-0.028908	-	0.164189	-
	Li's method						
	1 vs. 2 vs. 3 (RI_m)	0.219802	0.228033	-0.037450	0.055378	0.052334	0.961
	1 vs. 2 vs. 3 (NRI_m)	0.253724	0.262335	-0.033940	0.053136	0.055160	0.936
	Pencina's method						
	1 vs. 2/3 (NRI_b)	0.508253	0.510225	-0.003879	0.113851	0.081039	0.988
	1/2 vs. 3 (NRI_b)	0.313462	0.356179	-0.136277	0.133880	0.136713	0.931
	Nakas's method						
	1 vs.2 vs. 3 (ΔVUS)	-	0.259338	-	-	0.006290	-
	Delong's method						
	1 vs. 2/3 (ΔAUC)	0.254127	0.255112	-0.003878	0.038315	0.040519	0.929
	1/2 vs. 3 (ΔAUC)	0.156731	0.178090	-0.136280	0.052752	0.068357	0.848
n=300	Proposed method						
	1 vs.2 vs. 3 (NRI_{o,w_1})	0.704711	0.718332	-0.019328	0.116569/0.066742	0.083217	0.991/0.881
	1 vs.2 vs. 3 (NRI_{o,w_2})	1.034413	1.056447	-0.021301	0.171616/0.103699	0.134244	0.986/0.856
	1 vs.2 vs. 3 (NRI_{o,w_3})	0.829152	0.850615	-0.025886	-	0.110412	-
	Li's method						
	1 vs. 2 vs. 3 (RI_m)	0.219802	0.225410	-0.025516	0.039208	0.036168	0.968
	1 vs. 2 vs. 3 (NRI_m)	0.253724	0.259676	-0.023460	0.037665	0.037957	0.951
	Pencina's method						
	1 vs. 2/3 (NRI_b)	0.508253	0.508845	-0.001164	0.080557	0.057975	0.987
	1/2 vs. 3 (NRI_b)	0.313462	0.331179	-0.056522	0.092032	0.099999	0.925
	Nakas's method						
	1 vs.2 vs. 3 (ΔVUS)	-	0.258253	-	-	0.042383	-
	Delong's method						
	1 vs. 2/3 (ΔAUC)	0.254127	0.254423	-0.001166	0.026677	0.028988	0.925
	1/2 vs. 3 (ΔAUC)	0.156731	0.165589	-0.056519	0.036937	0.049999	0.839

† Prevalence of outcome -1: 61.88%, 2: 24.11%, 3: 14.01%

Table 17. Simulation results based on an ordinal logistic structure – scenario 12

	Outcome†	θ	Avg. $\hat{\theta}$	Relative bias	Avg. $SE_{\hat{\theta}}$	$SD_{\hat{\theta}}$	CR
n=150	Proposed method						
	1 vs.2 vs. 3 (NRI_{o,w_1})	0.776085	0.770817	0.006787	0.165836/0.090836	0.115095	0.987/0.863
	1 vs.2 vs. 3 (NRI_{o,w_2})	1.154578	1.151844	0.002368	0.237909/0.137441	0.186875	0.984/0.839
	1 vs.2 vs. 3 (NRI_{o,w_3})	0.905450	0.905866	-0.000459	-	0.160271	-
	Li's method						
	1 vs. 2 vs. 3 (RI_m)	0.252329	0.254022	-0.006711	0.056471	0.050735	0.969
	1 vs. 2 vs. 3 (NRI_m)	0.286632	0.289931	-0.011511	0.053897	0.052539	0.953
	Pencina's method						
	1 vs. 2/3 (NRI_b)	0.492762	0.495425	-0.005405	0.110797	0.094657	0.978
	1/2 vs. 3 (NRI_b)	0.395110	0.433954	-0.098313	0.140924	0.126697	0.953
	Nakas's method						
	1 vs.2 vs. 3 (ΔVUS)	-	0.286617	-	-	0.061431	-
	Delong's method						
	1 vs. 2/3 (ΔAUC)	0.246381	0.247713	-0.005407	0.041385	0.047328	0.908
	1/2 vs. 3 (ΔAUC)	0.197555	0.216977	-0.098314	0.051771	0.063349	0.852
n=300	Proposed method						
	1 vs.2 vs. 3 (NRI_{o,w_1})	0.776085	0.774802	0.001653	0.117302/0.064537	0.080063	0.992/0.882
	1 vs.2 vs. 3 (NRI_{o,w_2})	1.154578	1.154919	-0.000295	0.167724/0.097010	0.130250	0.984/0.859
	1 vs.2 vs. 3 (NRI_{o,w_3})	0.905450	0.908769	-0.003665	-	0.110965	-
	Li's method						
	1 vs. 2 vs. 3 (RI_m)	0.252329	0.253405	-0.004265	0.039981	0.035236	0.972
	1 vs. 2 vs. 3 (NRI_m)	0.286632	0.288703	-0.007226	0.038240	0.036084	0.959
	Pencina's method						
	1 vs. 2/3 (NRI_b)	0.492762	0.493908	-0.002326	0.078506	0.071876	0.965
	1/2 vs. 3 (NRI_b)	0.395110	0.415718	-0.052159	0.097838	0.091497	0.956
	Nakas's method						
	1 vs.2 vs. 3 (ΔVUS)	-	0.286931	-	-	0.042681	-
	Delong's method						
	1 vs. 2/3 (ΔAUC)	0.246381	0.246954	-0.002326	0.029422	0.035938	0.893
	1/2 vs. 3 (ΔAUC)	0.197555	0.207859	-0.052159	0.036507	0.045749	0.860

† Prevalence of outcome -1: 60.96%, 2: 23.22%, 3: 15.82%

Table 18. Simulation results based on an ordinal logistic structure – scenario 13

	Outcome†	θ	Avg. $\hat{\theta}$	Relative bias	Avg. $SE_{\hat{\theta}}$	$SD_{\hat{\theta}}$	CR
n=150	Proposed method						
	1 vs.2 vs. 3 (NRI_{o,w_1})	1.658530	1.701680	-0.026017	0.325733/0.082286	0.110247	1.000/0.805
	1 vs.2 vs. 3 (NRI_{o,w_2})	2.668095	2.894219	-0.084751	0.554685/0.225542	0.285009	1.000/0.761
	1 vs.2 vs. 3 (NRI_{o,w_3})	1.913388	2.026049	-0.058880	-	0.184646	-
	Li's method						
	1 vs. 2 vs. 3 (RI_m)	0.269217	0.318010	-0.181240	0.061763	0.050063	0.957
	1 vs. 2 vs. 3 (NRI_m)	0.286143	0.333275	-0.164716	0.060878	0.049992	0.957
	Pencina's method						
	1 vs. 2/3 (NRI_b)	0.638253	0.631799	0.010112	0.124745	0.079742	0.992
	1/2 vs. 3 (NRI_b)	0.708103	0.733258	-0.035520	0.178071	0.096479	0.998
	Nakas's method						
	1 vs.2 vs. 3 (ΔVUS)	-	-	-	-	-	-
	Delong's method						
	1 vs. 2/3 (ΔAUC)	0.319127	0.315900	0.010111	0.036357	0.039871	0.931
	1/2 vs. 3 (ΔAUC)	0.354052	0.366629	-0.035524	0.043576	0.048240	0.874
n=300	Proposed method						
	1 vs.2 vs. 3 (NRI_{o,w_1})	1.658530	1.690615	-0.019345	0.224958/0.054423	0.071387	1.000/0.805
	1 vs.2 vs. 3 (NRI_{o,w_2})	2.668095	2.809004	-0.052813	0.377538/0.152567	0.193327	1.000/0.793
	1 vs.2 vs. 3 (NRI_{o,w_3})	1.913388	1.974821	-0.032107	-	0.126591	-
	Li's method						
	1 vs. 2 vs. 3 (RI_m)	0.269217	0.298233	-0.107778	0.041583	0.034119	0.955
	1 vs. 2 vs. 3 (NRI_m)	0.286143	0.312938	-0.093643	0.040994	0.034093	0.954
	Pencina's method						
	1 vs. 2/3 (NRI_b)	0.638253	0.635906	0.003678	0.088330	0.051763	0.996
	1/2 vs. 3 (NRI_b)	0.708103	0.720131	-0.016986	0.124346	0.066417	0.999
	Nakas's method						
	1 vs.2 vs. 3 (ΔVUS)	-	-	-	-	-	-
	Delong's method						
	1 vs. 2/3 (ΔAUC)	0.319127	0.317953	0.003678	0.025238	0.025882	0.935
	1/2 vs. 3 (ΔAUC)	0.354052	0.360066	-0.016987	0.031304	0.033208	0.916

† Prevalence of outcome -1: 55.82%, 2: 11.04%, 3: 16.66%, 4: 6.25%, 5: 9.86%

Table 19. Simulation results based on an ordinal logistic structure – scenario 14

	Outcome†	θ	Avg. $\hat{\theta}$	Relative bias	Avg. $SE_{\hat{\theta}}$	$SD_{\hat{\theta}}$	CR
n=150	Proposed method						
	1 vs.2 vs. 3 (NRI_{o,w_1})	1.658480	1.690799	-0.019487	0.314633/0.085966	0.119548	1.000/0.802
	1 vs.2 vs. 3 (NRI_{o,w_2})	2.667892	2.882370	-0.080392	0.537804/0.226923	0.292459	1.000/0.768
	1 vs.2 vs. 3 (NRI_{o,w_3})	1.913337	2.014814	-0.053037	-	0.193925	-
	Li's method						
	1 vs. 2 vs. 3 (RI_m)	0.269217	0.317752	-0.180281	0.060083	0.050143	0.949
	1 vs. 2 vs. 3 (NRI_m)	0.286143	0.332951	-0.163584	0.059105	0.049833	0.949
	Pencina's method						
	1 vs. 2/3 (NRI_b)	0.638253	0.632413	0.009150	0.123665	0.084070	0.986
	1/2 vs. 3 (NRI_b)	0.708103	0.745314	-0.052550	0.174253	0.090890	0.999
	Nakas's method						
	1 vs.2 vs. 3 (ΔVUS)	-	-	-	-	-	-
	Delong's method						
	1 vs. 2/3 (ΔAUC)	0.319127	0.316207	0.009149	0.036906	0.042035	0.919
	1/2 vs. 3 (ΔAUC)	0.354052	0.372657	-0.052550	0.041596	0.045445	0.854
n=300	Proposed method						
	1 vs.2 vs. 3 (NRI_{o,w_1})	1.658480	1.688403	-0.018042	0.218524/0.055733	0.073321	1.000/0.823
	1 vs.2 vs. 3 (NRI_{o,w_2})	2.667892	2.815745	-0.055419	0.367654/0.151730	0.190747	1.000/0.774
	1 vs.2 vs. 3 (NRI_{o,w_3})	1.913337	1.974079	-0.031747	-	0.126399	-
	Li's method						
	1 vs. 2 vs. 3 (RI_m)	0.269217	0.300625	-0.116660	0.040523	0.033261	0.948
	1 vs. 2 vs. 3 (NRI_m)	0.286143	0.315143	-0.101350	0.039915	0.033300	0.947
	Pencina's method						
	1 vs. 2/3 (NRI_b)	0.638253	0.639022	-0.001200	0.087751	0.054682	0.994
	1/2 vs. 3 (NRI_b)	0.708103	0.734295	-0.036990	0.121912	0.063449	1.000
	Nakas's method						
	1 vs.2 vs. 3 (ΔVUS)	-	-	-	-	-	-
	Delong's method						
	1 vs. 2/3 (ΔAUC)	0.319127	0.319511	-0.001200	0.025546	0.027341	0.934
	1/2 vs. 3 (ΔAUC)	0.354053	0.367147	-0.036990	0.029832	0.031724	0.886

† Prevalence of outcome -1: 55.60%, 2: 10.64%, 3: 16.30%, 4: 6.64%, 5: 10.83%

Figure 1. Scatter plot for scenario1

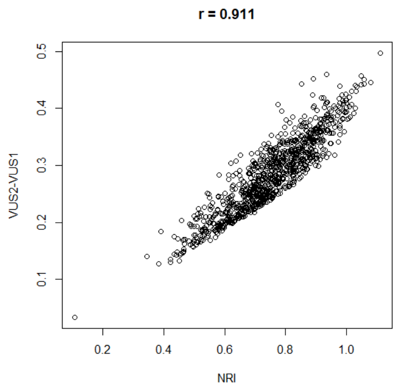
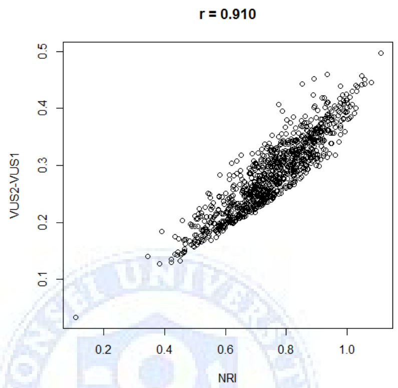
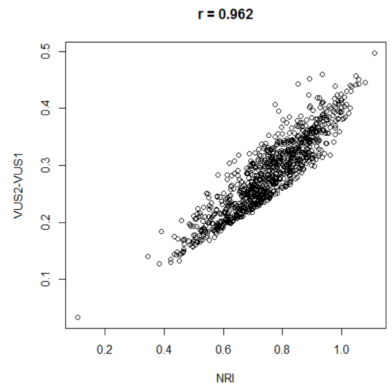
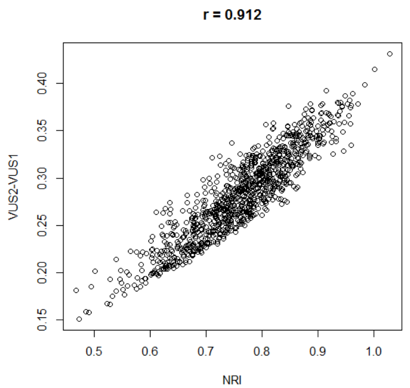
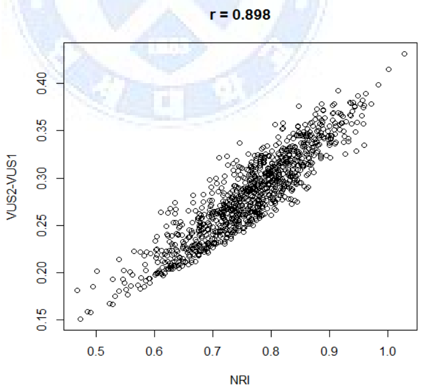
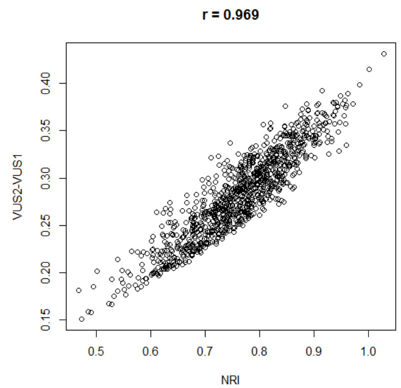
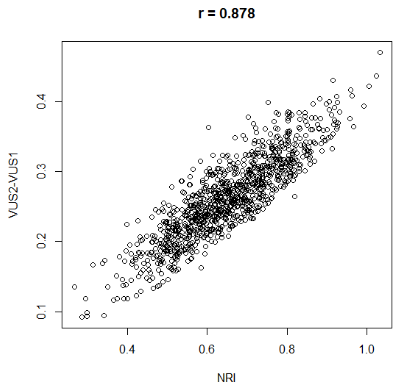
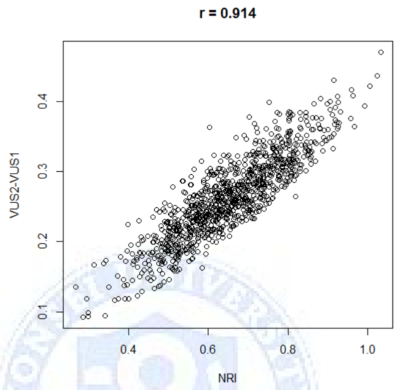
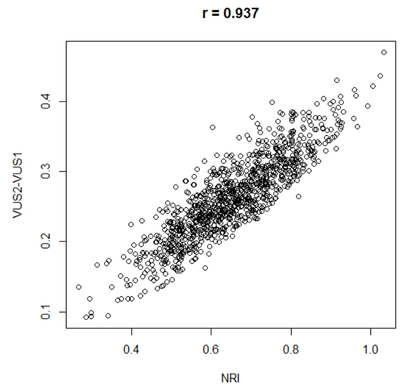
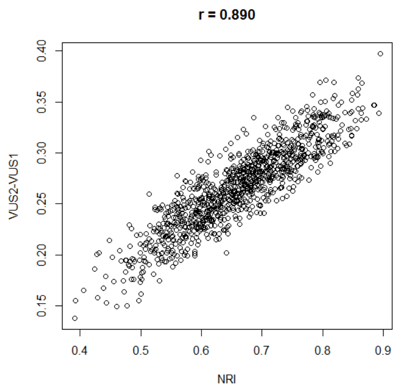
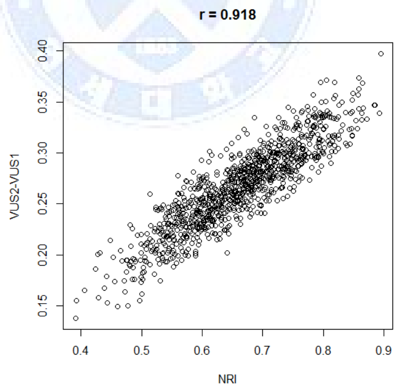
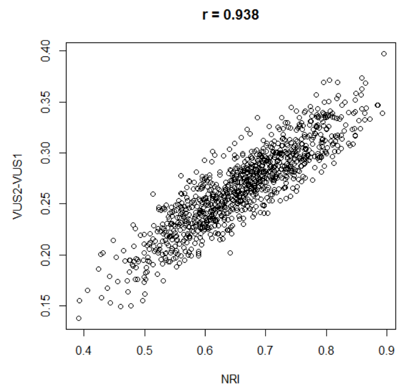
	NRI_{o,w_1} vs. ΔVUS	NRI_{o,w_2} vs. ΔVUS	NRI_{o,w_3} vs. ΔVUS
n=150	 Correlation coefficient r=0.911	 Correlation coefficient r=0.910	 Correlation coefficient r=0.962
n=300	 Correlation coefficient r=0.912	 Correlation coefficient r=0.898	 Correlation coefficient r=0.969

Figure 2. Scatter plot for scenario8

	NRI_{o,w_1} vs. ΔVUS	NRI_{o,w_2} vs. ΔVUS	NRI_{o,w_3} vs. ΔVUS
n=150	 Correlation coefficient r=0.878	 Correlation coefficient r=0.914	 Correlation coefficient r=0.937
n=300	 Correlation coefficient r=0.890	 Correlation coefficient r=0.918	 Correlation coefficient r=0.938

V. Application

5.1 Glaucoma data (three-category outcome)

Glaucoma affects over 70 million people worldwide, (Quigley, 1996; Resnikoff, 2004) and is the second most frequent cause of blindness. Glaucoma is usually diagnosed as normal, mild, and moderate to severe. The average peripapillary retinal nerve fiber layer thickness of each patients is obtained with swept-source optical coherence tomography and the thickness is measured in four modes: temporal, superior, nasal and inferior. Among the four different modes, the present study used the temporal and superior modes to generate two models in which the superior mode was added to the first model comparing only temporal mode data. Subsequently, the predictive power of the models was assessed. For better understanding, Table 20 lists the data used for the analysis.

While keeping the outcomes as ordinal data, the outcomes were classified into binary outcomes with arbitrary cutoff points. The two models are described below:

M_1: age + gender + temporal mode,

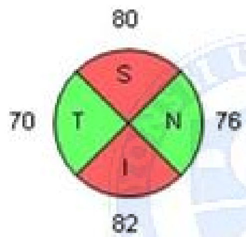
M_2: age + gender + temporal mode + superior mode.

The results of logistic regression obtained from the two models are listed in Table 21. All the results in the second model with the added variable (superior mode) were statistically significant.

The results in Table 22 show the statistical significance of NRI and the SE obtained from the newly proposed method and a bootstrap method, in comparison to the other methods.

When considering the outcomes as ordinal data, the predictive ability of glaucomatous eyes measured by VUS (from 0.291 to 0.497; p-value <0.001) and NRI (NRI=0.453; p-value<0.001, p-value=0.002, NRI=0.697; p-value<0.001, p-value=0.003, NRI=0.556; p-value=0.005) improved by adding superior thickness to other clinical predictors (age, gender, temporal mode). When considering outcomes as multiclass data, the predictive ability of glaucomatous eyes measured by RI (RI=0.163, p-value=0.001, p-value=0.010) and NRI (NRI=0.192, p-value<0.001, p-value<0.001) improved with the addition. When considering outcomes as binary data of 1 vs. 2/3, the predictive ability of glaucomatous eyes measured by AUC (from 0.605 to 0.697; p-value=0.013, p-value =0.046) and NRI (NRI=0.284; p-value =0.001, p-value =0.002) improved with the addition. When considering outcomes as binary data of 1/2 vs. 3, the predictive ability for glaucomatous eyes measured by AUC (from 0.665 to 772; p-value=0.096, p-value=0.010) improved with adding superior thickness to other clinical predictors, while that measured by NRI showed no improvement (NRI=0.198; p-value=0.071, p-value=0.139).

Table 20. Description of glaucoma data

<table>
<tr><th>Variables</th><th>Type of variables</th><th>Explanation for variables</th></tr>
<tr><td>Glaucoma severity</td><td>ordinal</td><td>1= normal (n=91), 2= mild glaucoma (n=32), 3= moderate to severe glaucoma (n=26)</td></tr>
<tr><td>Age</td><td>continuous</td><td>23 ~ 83</td></tr>
<tr><td>Gender</td><td>nominal</td><td>1: male, 2: female</td></tr>
<tr><td>Temporal mode</td><td>continuous</td><td rowspan="4">


RNLF thickness which is taken Swept source Optical Coherence Tomography (SS OCT) is consist of four modes (temporal, superior, nasal, inferior).</td></tr>
<tr><td>Superior mode</td><td>continuous</td></tr>
<tr><td>Nasal mode</td><td>continuous</td></tr>
<tr><td>Inferior mode</td><td>continuous</td></tr>
</table>

Table 21. Result of ordinal logistic regression for glaucoma data

	Model1		Model2	
	OR (95% CI)	p-value	OR(95% CI)	p-value
Ordinal logistic regression				
Age	0.992 (0.967-1.017)	0.5147	0.999 (0.971-1.028)	0.9325
Gender				
Male	Ref		Ref	
Female	0.575 (0.286-1.158)	0.1212	0.789 (0.359-1.736)	0.5559
Temporal mode	0.932 (0.907-0.957)	<.0001	0.961 (0.933-0.990)	0.0089
Superior mode			0.940 (0.923-0.959)	<.0001
Binary logistic regression (1 vs. 2/3)				
Age	0.993 (0.967-1.019)	0.5899	0.998 (0.967-1.030)	0.8977
Gender				
Male	Ref		Ref	
Female	0.630 (0.300-1.325)	0.2235	0.874 (0.353-2.165)	0.7705
Temporal mode	0.936 (0.909-0.963)	<.0001	0.955 (0.922-0.990)	0.0110
Superior mode			0.935 (0.913-0.959)	<.0001
Binary logistic regression (1/2 vs. 3)				
Age	0.992 (0.958-1.028)	0.6705	1.004 (0.961-1.048)	0.8735
Gender				
Male	Ref			
Female	0.374 (0.142-0.990)	0.0477	0.470 (0.154-1434)	0.1844
Temporal mode	0.921 (0.887-0.957)	<.0001	0.972 (0.933-1.013)	0.1784
Superior mode			0.944 (0.920-0.969)	<.0001

Table 22. Result of predictive ability for glaucoma data

Outcome	$\hat{\theta}$	$SE_{\hat{\theta}}$	p-value	$SE_{\hat{\theta},boot}$	p-value
Proposed method					
1 vs.2 vs. 3 (NRI_{o,w_1})	0.453	0.118/0.102	<0.001/<0.001	0.145	0.002
1 vs.2 vs. 3 (NRI_{o,w_2})	0.697	0.158/0.145	<0.001/<0.001	0.234	0.003
1 vs.2 vs. 3 (NRI_{o,w_3})	0.556			0.203	0.005
Li's method					
1 vs. 2 vs. 3 (RI_m)	0.163	0.048	0.001	0.064	0.010
1 vs. 2 vs. 3 (NRI_m)	0.192	0.043	<0.001	0.052	<0.001
Pencina's method					
1 vs. 2/3 (NRI_b)	0.284	0.083	0.001	0.093	0.002
1/2 vs. 3 (NRI_b)	0.198	0.110	0.071	0.134	0.139
Nakas's method					
1 vs.2 vs. 3 (ΔVUS)	0.206 (0.497-0.291)	-	-	0.026	<0.001
Delong's method					
1 vs. 2/3 (ΔAUC)	0.092 (0.697-0.605)	0.037	0.013	0.046	0.046
1/2 vs. 3 (ΔAUC)	0.107 (0.772-0.665)	0.064	0.096	0.065	0.101

5.2 Nonrelevant cerebral atherosclerosis data (seven-category outcome)

Acute cerebral infarction is a type of ischemic stroke resulting from a blockage in the blood vessels supplying blood to the brain. Finding prognostic factors in patients with acute cerebral infraction is very important (Kim et al., 2013).

The modified Rankin scale (mRS) is often used as functional outcome of prognostic factors. mRS consists of ordinal categories, from 0 to 6, and with higher numbers representing more severe symptom. The meaning of each category is described in Table 23.

The factors in Table 24 (age, sex, National Institute of Health Stroke Scale score at admission [NIHSS], diabetes mellitus, peripheral artery occlusive disease, thrombolysis at admission, white blood cell count, platelet count, high-sensitivity C-reactive protein [hs-CRP], albumin, glucose, relevant cerebral atherosclerosis [RCA], current smoking, hemoglobin, triglycerides and blood urea nitrogen) are known to be prognostic factors of functional outcomes. Therefore, to determine whether the prognostic ability thereof can be improved with the addition of the presence of nonrelevant cerebral atherosclerosis (NCAR) or burden of nonrelevant cerebral atherosclerosis, the two models described below were devised.

M_1: Known prognostic factors (age + ~ + blood urea nitrogen),

M_2: Known prognostic factors (age + ~ + blood urea nitrogen) + presence of NCAR or burden of NCAR.

M_1 includes only well-known risk factors, while a new factor is added in M_2. There are two cases in M_2: one includes the presence of nonrelevant cerebral atherosclerosis and the other uses burden of nonrelevant cerebral atherosclerosis. For better understanding, the variables used in the analysis are described in Table 24, and the results are presented in Table 25.

The outcome variable originally had seven categories; however, we divided the outcome variable into two and seven categories arbitrarily to check for various peculiarities. Since the proportional odds assumption was satisfied, ordinal logistic regression was conducted in each model. The results showed significance in all outcome levels when the presence of nonrelevant cerebral atherosclerosis or burden of nonrelevant cerebral atherosclerosis was added to the model.

As in the results for NRI obtained from the newly proposed method, RI improved with the addition of the presence of nonrelevant cerebral atherosclerosis or burden of nonrelevant cerebral atherosclerosis, when the original seven categories were kept. The noted improvement in NRI was not observed for the other categories that were divided arbitrarily.

Table 23. Modified Rankin scale

mRS	Explanation
0	No symptom at all
1	No significant disability despite symptoms; able to carry out all usual duties and activities
2	Slight disability; unable to carry out all previous activities, but able to look after own affairs without assistance
3	Moderate disability; requiring some help, but able to walk without assistance;
4	Moderately severe disability; unable to walk without assistance, and unable to attend to own bodily needs without assistance
5	Severe disability; bed-ridden, incontinence and requiring constant nursing care and attention
6	Dead

Table 24. Description of nonrelevant cerebral atherosclerosis data

Variables	Type of variables	Explanation for variables
mRS	ordinal	0-6
Age	continuous	
Sex	nominal	male=1, female=0, binary
NIHSS score at admission	continuous	0-46
Diabetes mellitus	nominal	0=absent, 1=present
Peripheral artery occlusive disease	nominal	0=absent, 1=present
Thrombolysis at admission	nominal	0=not done, 1= done
White blood cell count	continuous	$x10^9/L$
Platelet count	continuous	$x10^9/L$
hs-CRP	continuous	nmol/L
Albumin	continuous	g/dL
Glucose	continuous	mmol/L
RCA	nominal	0=absent, 1=present
Current smoking	nominal	0=absent, 1=present
Hemoglobin	continuous	g/dL
Triglyceride	continuous	mmol/L
Blood urea nitrogen	continuous	mmol/L
Presence of nonrelevant cerebral atherosclerosis	nominal	0=absent, 1=present
Burden of nonrelevant cerebral atherosclerosis	continuous	0~

Table 25. Results of ordinal logistic regression for nonrelevant cerebral atherosclerosis data

	Outcome level‡	OR(95% CI)	p-value
Presence of nonrelevant cerebral atherosclerosis			
Model1†	7 (0 vs. 1 vs. 2 vs. 3 vs. 4 vs. 5 vs. 6)	1.511(1.128-2.024)	0.0056
Model2†	6 (0/1 vs. 2 vs. 3 vs. 4 vs. 5 vs. 6)	1.697(1.199-2.403)	0.0029
Model3†	5 (0/1 vs. 2 vs. 3/4 vs. 5 vs. 6)	1.714(1.207-2.434)	0.0026
Model4†	4 (0/1 vs. 2 vs. 3/4 vs. 5/6)	1.684(1.182-2.399)	0.0039
Model5†	3 (0/1 vs. 2/3 vs. 4/5/6)	1.608(1.116-2.320)	0.0108
Model6†	3 (0/1/2 vs. 3/4 vs. 5/6)	2.131(1.343-3.383)	0.0013
Model7†	2 (0/1 vs. 2/3/4/5/6)	1.573(1.054-2.348)	0.0265
Model8†	2 (0/1/2 vs. 3/4/5/6)	2.296(1.393-3.785)	0.0011
Burden of nonrelevant cerebral atherosclerosis			
Model1†	7 (0 vs. 1 vs. 2 vs. 3 vs. 4 vs. 5 vs. 6)	1.329(1.130-1.563)	0.0006
Model2†	6 (0/1 vs. 2 vs. 3 vs. 4 vs. 5 vs. 6)	1.309(1.157-1.669)	0.0004
Model3†	5 (0/1 vs. 2 vs. 3/4 vs. 5 vs. 6)	1.417(1.177-1.706)	0.0002
Model4†	4 (0/1 vs. 2 vs. 3/4 vs. 5/6)	1.415(1.173-1.707)	0.0003
Model5†	3 (0/1 vs. 2/3 vs. 4/5/6)	1.310(1.075-1.596)	0.0074
Model6†	3 (0/1/2 vs. 3/4 vs. 5/6)	1.635(1.301-2.055)	<.0001
Model7†	2 (0/1 vs. 2/3/4/5/6)	1.281(1.026-1.599)	0.0286
Model8†	2 (0/1/2 vs. 3/4/5/6)	1.676(1.298-2.164)	<.0001

† Adjusted age, sex, NIHSS score at admission, diabetes mellitus, peripheral artery occlusive disease, thrombolysis at admission, white blood cell count, platelet count, hs-CRP, albumin, glucose, RCA, current smoking, hemoglobin, triglyceride and blood urea nitrogen

‡ n=749, 0: 216(28.84%), 1: 291(38.85%), 2: 106(14.15%), 3: 44(5.87%), 4: 60(8.01%), 5: 17(2.27%), 6: 15(2.00%)

Table 26. Result of NRI based on ordinal logistic regression for nonrelevant cerebral atherosclerosis data

	Presence of nonrelevant cerebral atherosclerosis					Burden of nonrelevant cerebral atherosclerosis				
	NRI	SE_{SM}	p-value	SE_B	p-value	NRI	SE_{SM}	p-value	SE_B	p-value
Model1										
w_1	0.038	0.012	0.002	0.012	0.001	0.088	0.032	0.006	0.031	0.005
w_2	0.134	0.043	0.002	0.043	0.002	0.229	0.091	0.012	0.088	0.009
w_3	0.129	-	-	-	-	0.211	-	-	-	-
Model2										
w_1	0.040	0.039	0.315	0.038	0.301	0.075	0.048	0.115	0.047	0.108
w_2	0.042	0.041	0.301	0.040	0.288	0.139	0.090	0.124	0.088	0.115
w_3	0.041	-	-	-	-	0.124	-	-	-	-
Model3										
w_1	0.010	0.046	0.837	0.045	0.832	0.027	0.050	0.588	0.049	0.587
w_2	-0.020	0.094	0.829	0.091	0.824	0.067	0.124	0.588	0.124	0.587
w_3	-0.010	-	-	-	-	0.054	-	-	-	-
Model4										
w_1	0.031	0.039	0.436	0.039	0.436	0.021	0.402	0.602	0.402	0.602
w_2	0.047	0.053	0.380	0.053	0.380	0.093	0.063	0.142	0.063	0.142
w_3	0.022	-	-	-	-	0.078	-	-	-	-
Model5										
w_1	-0.001	0.020	0.971	0.020	0.971	0.010	0.021	0.626	0.021	0.626
w_2	-0.012	0.036	0.732	0.036	0.732	0.015	0.035	0.668	0.035	0.667
w_3	-0.012	-	-	-	-	0.015	-	-	-	-
Model6										
w_1	0.010	0.033	0.774	0.033	0.774	0.020	0.038	0.589	0.038	0.541
w_2	0.034	0.073	0.642	0.073	0.639	0.040	0.081	0.619	0.080	0.618
w_3	0.034	-	-	-	-	0.040	-	-	-	-
Model7										
w_1	-0.002	0.014	0.618	-	-	0.004	0.013	0.328	-	-
Model8										
w_1	-0.005	0.024	0.618	-	-	-0.016	0.025	0.531	-	-

VI. Discussion and conclusion

Net reclassification improvement is often used to examine discrimination ability when a new factor is added to known risk factors. However, no method of using NRI to evaluate ordinal outcomes has been developed.

NRI for binary outcomes was suggested by Pencina et al. (2008), and NRI for multiclass outcomes, as suggested by Li et al. (2013), was extended considering the meaning of order. Moreover, variation estimation was suggested with the Stuart-Maxwell test and Bhapkar's test statistics (Stuart, 1955; Maxwell, 1970; Sun et al., 2008). Therefore, the present study attempted to propose a method of using NRI for ordinal outcomes considering weights. Weights take into account closeness to the true category when counting reclassification. The present study considered three different weights: w_1 considered only closeness from the true category, while w_2 and w_3 considered not only closeness from the true category but also the correct classification for $C(M_1)$ and $C(M_2)$ from the true category.

In comparison of known methods to the new method proposed herein through simulation study, coverage rates for the new method were generally higher than those for other methods in multinomial settings and ordinal settings. VUS, a measurement for checking the predictive ability of ordinal data, has the disadvantage of a long computing time for large sample sizes or for numerous outcome categories. Meanwhile, however, NRI has a relatively shorter computing time. Notwithstanding, the proposed method tends to over-estimate when another factor is added or the number of outcome categories increases, and thus careful attention to interpretation is needed.

The noted methods were applied to real data to validate the study results. When the ordinal outcomes were divided arbitrarily, the predictive ability with the addition of new factors was not improved using existing method; however, when the original categories were kept, NRI using the newly proposed method revealed improved predictive ability with the addition of new factors to existing known factors. Even though the same ordinal data were used, the present study demonstrated that the discrimination of new factors using original category outcomes is better than using category outcomes divided arbitrarily.

For further study, studies on methods for estimating variance with no independent assumption and on integrated discrimination improvement for ordinal outcomes are needed. Simulation studies for various prevalence rates are also required for generalizing results.

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Supplementary materials

Appendix S1: Reclassification table with weight w_1 for ordinal outcomes by M_1 and M_2

1. Two-category outcome

$n_{cC(M_1)C(M_2)}$

$w_{cC(M_1)C(M_2)}$

Non-event

c =1

$C(M_1) \setminus C(M_2)$	1	2	
1	n_{111}	n_{112}	n_{11+}
2	n_{121}	n_{212}	n_{12+}
	n_{1+1}	n_{1+2}	n_{1++}

	1	2	
1		-1	
2	1		

↓

c =2

Event

$C(M_1) \setminus C(M_2)$	1	2	
1	n_{211}	n_{212}	n_{21+}
2	n_{221}	n_{222}	n_{22+}
	n_{2+1}	n_{2+2}	n_{2++}

	1	2	
1		1	
2	-1		

2. Three-category outcome

$n_{cC(M_1)C(M_2)}$ $w_{cC(M_1)C(M_2)}$

Non-event

c =1

$C(M_1) \setminus C(M_2)$	1	2	3	
1	n_{111}	n_{112}	n_{113}	n_{11+}
2	n_{121}	n_{122}	n_{123}	n_{12+}
3	n_{131}	n_{132}	n_{133}	n_{13+}
	n_{1+1}	n_{1+2}	n_{1+3}	n_{1++}

	1	2	3	
1		-1/2	-2/2	
2	1/2		-1/2	
3	2/2	1/2		

↓

c =2

$C(M_1) \setminus C(M_2)$	1	2	3	
1	n_{211}	n_{212}	n_{213}	n_{21+}
2	n_{221}	n_{222}	n_{223}	n_{22+}
3	n_{231}	n_{232}	n_{233}	n_{23+}
	n_{2+1}	n_{2+2}	n_{2+3}	n_{2++}

	1	2	3	
1		1/2		
2	-1/2		-1/2	
3		1/2		

Event

c =3

$C(M_1) \setminus C(M_2)$	1	2	3	
1	n_{311}	n_{312}	n_{313}	n_{31+}
2	n_{321}	n_{322}	n_{323}	n_{32+}
3	n_{331}	n_{332}	n_{333}	n_{33+}
	n_{3+1}	n_{3+2}	n_{3+3}	n_{3++}

	1	2	3	
1		1/2	2/2	
2	-1/2		1/2	
3	-2/2	-1/2		

3. Four-category outcome

$n_{cC(M_1)C(M_2)}$ $w_{cC(M_1)C(M_2)}$

Non-event

↓

c =1

$C(M_1)$ \ $C(M_2)$	1	2	3	4	
1	n_{111}	n_{112}	n_{113}	n_{114}	n_{11+}
2	n_{121}	n_{122}	n_{123}	n_{124}	n_{12+}
3	n_{131}	n_{132}	n_{133}	n_{134}	n_{13+}
4	n_{141}	n_{142}	n_{143}	n_{144}	n_{14+}
	n_{1+1}	n_{1+2}	n_{1+3}	n_{1+4}	n_{1++}

	1	2	3	4	
1		-1/3	-2/3	-3/3	
2	1/3		-1/3	-2/3	
3	2/3	1/3		-1/3	
4	3/3	2/3	1/3		

c =2

$C(M_1)$ \ $C(M_2)$	1	2	3	4	
1	n_{211}	n_{212}	n_{213}	n_{214}	n_{21+}
2	n_{221}	n_{222}	n_{223}	n_{224}	n_{22+}
3	n_{231}	n_{232}	n_{233}	n_{234}	n_{23+}
4	n_{241}	n_{242}	n_{243}	n_{244}	n_{24+}
	n_{2+1}	n_{2+2}	n_{2+3}	n_{2+4}	n_{2++}

	1	2	3	4	
1		1/3		-1/3	
2	-1/3		-1/3	-2/3	
3		1/3		-1/3	
4	1/3	2/3	1/3		

c =3

$C(M_1)$ \ $C(M_2)$	1	2	3	4	
1	n_{311}	n_{312}	n_{313}	n_{314}	n_{31+}
2	n_{321}	n_{322}	n_{323}	n_{324}	n_{32+}
3	n_{331}	n_{332}	n_{333}	n_{334}	n_{33+}
4	n_{341}	n_{342}	n_{343}	n_{344}	n_{34+}
	n_{3+1}	n_{3+2}	n_{3+3}	n_{3+4}	n_{3++}

	1	2	3	4	
1		1/3	2/3	1/3	
2	-1/3		1/3		
3	-2/3	-1/3		-1/3	
4	-1/3		1/3		

Event

c =4

$C(M_1)$ \ $C(M_2)$	1	2	3	4	
1	n_{411}	n_{412}	n_{413}	n_{414}	n_{41+}
2	n_{421}	n_{422}	n_{423}	n_{424}	n_{42+}
3	n_{431}	n_{432}	n_{433}	n_{434}	n_{43+}
4	n_{441}	n_{442}	n_{443}	n_{444}	n_{44+}
	n_{4+1}	n_{4+2}	n_{4+3}	n_{4+4}	n_{4++}

	1	2	3	4	
1		1/3	2/3	3/3	
2	-1/3		1/3	2/3	
3	-2/3	-1/3		1/3	
4	-1	-2/3	-1/3		

4. Five-category outcome

$n_{cC(M_1)C(M_2)}$ $\qquad$ $w_{cC(M_1)C(M_2)}$

Non-event ↓

c =1

$C(M_1) \setminus C(M_2)$	1	2	3	4	5	
1	n_{111}	n_{112}	n_{113}	n_{114}	n_{115}	n_{11+}
2	n_{121}	n_{122}	n_{123}	n_{124}	n_{125}	n_{12+}
3	n_{131}	n_{132}	n_{133}	n_{134}	n_{135}	n_{13+}
4	n_{141}	n_{142}	n_{143}	n_{144}	n_{145}	n_{14+}
5	n_{151}	n_{152}	n_{153}	n_{154}	n_{155}	n_{15+}
	n_{1+1}	n_{1+2}	n_{1+3}	n_{1+4}	n_{1+5}	n_{1++}

	1	2	3	4	5
1		-1/4	-2/4	-3/4	-4/4
2	1/4		-1/4	-2/4	-3/4
3	2/4	1/4		-1/4	-2/4
4	3/4	2/4	1/4		-1/4
5	4/4	3/4	2/4	1/4	

c =2

$C(M_1) \setminus C(M_2)$	1	2	3	4	5	
1	n_{211}	n_{212}	n_{213}	n_{214}	n_{215}	n_{21+}
2	n_{221}	n_{222}	n_{223}	n_{224}	n_{225}	n_{22+}
3	n_{231}	n_{232}	n_{233}	n_{234}	n_{235}	n_{23+}
4	n_{241}	n_{242}	n_{243}	n_{244}	n_{245}	n_{24+}
5	n_{251}	n_{252}	n_{253}	n_{254}	n_{255}	n_{25+}
	n_{2+1}	n_{2+2}	n_{2+3}	n_{2+4}	n_{2+5}	n_{2++}

	1	2	3	4	5
1		1/4		-1/4	-2/4
2	-1/4		-1/4	-2/4	-3/4
3		1/4		-1/4	-2/4
4	1/4	2/4	1/4		-1/4
5	2/4	3/4	2/4	1/4	

c =3

$C(M_1) \setminus C(M_2)$	1	2	3	4	5	
1	n_{311}	n_{312}	n_{313}	n_{314}	n_{315}	n_{31+}
2	n_{321}	n_{322}	n_{323}	n_{324}	n_{325}	n_{32+}
3	n_{331}	n_{332}	n_{333}	n_{334}	n_{335}	n_{33+}
4	n_{341}	n_{342}	n_{343}	n_{344}	n_{345}	n_{34+}
5	n_{351}	n_{352}	n_{353}	n_{354}	n_{355}	n_{35+}
	n_{3+1}	n_{3+2}	n_{3+3}	n_{3+4}	n_{3+5}	n_{3++}

	1	2	3	4	5
1		1/4	2/4	1/4	
2	-1/4		1/4		-1/4
3	-2/4	-1/4		-1/4	-2/4
4	-1/4		1/4		-1/4
5		1/4	2/4	1/4	

c =4

$C(M_1) \setminus C(M_2)$	1	2	3	4	5	
1	n_{411}	n_{412}	n_{413}	n_{414}	n_{415}	n_{41+}
2	n_{421}	n_{422}	n_{423}	n_{424}	n_{425}	n_{42+}
3	n_{431}	n_{432}	n_{433}	n_{434}	n_{435}	n_{43+}
4	n_{441}	n_{442}	n_{443}	n_{444}	n_{445}	n_{44+}
5	n_{451}	n_{452}	n_{453}	n_{454}	n_{455}	n_{45+}
	n_{4+1}	n_{4+2}	n_{4+3}	n_{4+4}	n_{4+5}	n_{4++}

	1	2	3	4	5
1		1/4	2/4	3/4	2/4
2	-1/4		1/4	2/4	1/4
3	-2/4	-1/4		1/4	
4	-3/4	-2/4	-1/4		-1/4
5	-2/4	-1/4		1/4	

c =5

Event $C(M_1) \setminus C(M_2)$	1	2	3	4	5	
1	n_{511}	n_{512}	n_{513}	n_{514}	n_{515}	n_{51+}
2	n_{521}	n_{522}	n_{523}	n_{524}	n_{525}	n_{52+}
3	n_{531}	n_{532}	n_{533}	n_{534}	n_{535}	n_{53+}
4	n_{541}	n_{542}	n_{543}	n_{544}	n_{545}	n_{54+}
5	n_{551}	n_{552}	n_{553}	n_{554}	n_{555}	n_{55+}
	n_{5+1}	n_{5+2}	n_{5+3}	n_{5+4}	n_{5+5}	n_{5++}

	1	2	3	4	5	
1		1/4	2/4	3/4	4/4	
2	-1/4		1/4	2/4	3/4	
3	-2/4	-1/4		1/4	2/4	
4	-3/4	-2/4	-1/4		1/4	
5	-4/4	-3/4	-2/4	-1/4		

5. Six-category outcome

$n_{cC(M_1)C(M_2)}$ $\quad$ $w_{cC(M_1)C(M_2)}$

Non-event

c =1

$C(M_1)$ \ $C(M_2)$	1	2	3	4	5	6	
1	n_{111}	n_{112}	n_{113}	n_{114}	n_{115}	n_{116}	n_{11+}
2	n_{121}	n_{122}	n_{123}	n_{124}	n_{125}	n_{126}	n_{12+}
3	n_{131}	n_{132}	n_{133}	n_{134}	n_{135}	n_{136}	n_{13+}
4	n_{141}	n_{142}	n_{143}	n_{144}	n_{145}	n_{146}	n_{14+}
5	n_{151}	n_{152}	n_{153}	n_{154}	n_{155}	n_{156}	n_{15+}
6	n_{161}	n_{162}	n_{163}	n_{164}	n_{165}	n_{166}	n_{16+}
	n_{1+1}	n_{1+2}	n_{1+3}	n_{1+4}	n_{1+5}	n_{1+6}	n_{1++}

	1	2	3	4	5	6
1		-1/5	-2/5	-3/5	-4/5	-5/5
2	1/5		-1/5	-2/5	-3/5	-4/5
3	2/5	1/5		-1/5	-2/5	-3/5
4	3/5	2/5	1/5		-1/5	-2/5
5	4/5	3/5	2/5	1/5		-1/5
6	5/5	4/5	3/5	2/5	1/5	

c =2

$C(M_1)$ \ $C(M_2)$	1	2	3	4	5	6	
1	n_{211}	n_{212}	n_{213}	n_{214}	n_{215}	n_{216}	n_{21+}
2	n_{221}	n_{222}	n_{223}	n_{224}	n_{225}	n_{226}	n_{22+}
3	n_{231}	n_{232}	n_{233}	n_{234}	n_{235}	n_{236}	n_{23+}
4	n_{241}	n_{242}	n_{243}	n_{244}	n_{245}	n_{246}	n_{24+}
5	n_{251}	n_{252}	n_{253}	n_{254}	n_{255}	n_{256}	n_{25+}
6	n_{261}	n_{262}	n_{263}	n_{264}	n_{265}	n_{266}	n_{26+}
	n_{2+1}	n_{2+2}	n_{2+3}	n_{2+4}	n_{2+5}	n_{2+6}	n_{2++}

	1	2	3	4	5	6
1		1/5		-1/5	-2/5	-3/5
2	-1/5		-1/5	-2/5	-3/5	-4/5
3		1/5		-1/5	-2/5	-3/5
4	1/5	2/5	1/5		-1/5	-2/5
5	2/5	3/5	2/5	1/5		-1/5
6	3/5	4/5	3/5	2/5	1/5	

c =3

$C(M_1)$ \ $C(M_2)$	1	2	3	4	5	6	
1	n_{311}	n_{312}	n_{313}	n_{314}	n_{315}	n_{316}	n_{31+}
2	n_{321}	n_{322}	n_{323}	n_{324}	n_{325}	n_{326}	n_{32+}
3	n_{331}	n_{332}	n_{333}	n_{334}	n_{335}	n_{336}	n_{33+}
4	n_{341}	n_{342}	n_{343}	n_{344}	n_{345}	n_{346}	n_{34+}
5	n_{351}	n_{352}	n_{353}	n_{354}	n_{355}	n_{356}	n_{35+}
6	n_{361}	n_{362}	n_{363}	n_{364}	n_{365}	n_{366}	n_{36+}
	n_{3+1}	n_{3+2}	n_{3+3}	n_{3+4}	n_{3+5}	n_{3+6}	n_{3++}

	1	2	3	4	5	6
1		1/5	2/5	1/5		-1/5
2	-1/5		1/5		-1/5	-2/5
3	-2/5	-1/5		-1/5	-2/5	-3/5
4	-1/5		1/5		-1/5	-2/5
5		1/5	2/5	1/5		-1/5
6	1/5	2/5	3/5	2/5	1/5	

↓ c =4

$C(M_1)$ \ $C(M_2)$	1	2	3	4	5	6	
1	n_{411}	n_{412}	n_{413}	n_{414}	n_{415}	n_{416}	n_{41+}
2	n_{421}	n_{422}	n_{423}	n_{424}	n_{425}	n_{426}	n_{42+}
3	n_{431}	n_{432}	n_{433}	n_{434}	n_{435}	n_{436}	n_{43+}
4	n_{441}	n_{442}	n_{443}	n_{444}	n_{445}	n_{446}	n_{44+}
5	n_{451}	n_{452}	n_{453}	n_{454}	n_{455}	n_{456}	n_{45+}
6	n_{461}	n_{462}	n_{463}	n_{464}	n_{465}	n_{466}	n_{46+}
	n_{4+1}	n_{4+2}	n_{4+3}	n_{4+4}	n_{4+5}	n_{4+6}	n_{4++}

	1	2	3	4	5	6	
1		1/5	2/5	3/5	2/5	1/5	
2	-1/5		1/5	2/5	1/5		
3	-2/5	-1/5		1/5		-1/5	
4	-3/5	-2/5	-1/5		-1/5	-2/5	
5	-2/5	-1/5		1/5		-1/5	
6	-1/5		1/5	2/5	1/5		

c =5

$C(M_1)$ \ $C(M_2)$	1	2	3	4	5	6	
1	n_{511}	n_{512}	n_{513}	n_{514}	n_{515}	n_{516}	n_{51+}
2	n_{521}	n_{522}	n_{523}	n_{524}	n_{525}	n_{526}	n_{52+}
3	n_{531}	n_{532}	n_{533}	n_{534}	n_{535}	n_{536}	n_{53+}
4	n_{541}	n_{542}	n_{543}	n_{544}	n_{545}	n_{546}	n_{54+}
5	n_{551}	n_{552}	n_{553}	n_{554}	n_{555}	n_{556}	n_{55+}
6	n_{561}	n_{562}	n_{563}	n_{564}	n_{565}	n_{566}	n_{56+}
	n_{5+1}	n_{5+2}	n_{5+3}	n_{5+4}	n_{5+5}	n_{5+5}	n_{5++}

	1	2	3	4	5	6	
1		1/5	2/5	3/5	4/5	3/5	
2	-1/5		1/5	2/5	3/5	2/5	
3	-2/5	-1/5		1/5	2/5	1/5	
4	-3/5	-2/5	-1/5		1/5		
5	-4/5	-3/5	-2/5	-1/5		-1/5	
6	-3/5	-2/5	-1/5		1/5		

c =6

$C(M_1)$ \ $C(M_2)$	1	2	3	4	5	6	
1	n_{611}	n_{612}	n_{613}	n_{614}	n_{615}	n_{616}	n_{61+}
2	n_{621}	n_{622}	n_{623}	n_{624}	n_{625}	n_{626}	n_{62+}
3	n_{631}	n_{632}	n_{633}	n_{634}	n_{635}	n_{636}	n_{63+}
4	n_{641}	n_{642}	n_{643}	n_{644}	n_{645}	n_{645}	n_{64+}
5	n_{651}	n_{652}	n_{653}	n_{654}	n_{655}	n_{656}	n_{65+}
6	n_{661}	n_{662}	n_{663}	n_{664}	n_{665}	n_{666}	n_{66+}
	n_{6+1}	n_{6+2}	n_{6+3}	n_{6+4}	n_{6+5}	n_{6+5}	n_{6++}

	1	2	3	4	5	6	
1		1/5	2/5	3/5	4/5	5/5	
2	-1/5		1/5	2/5	3/5	4/5	
3	-2/5	-1/5		1/5	2/5	3/5	
4	-3/5	-2/5	-1/5		1/5	2/5	
5	-4/5	-3/5	-2/5	-1/5		1/5	
6	-5/5	-4/5	-3/5	-2/5	-1/5		

6. Seven-category outcome

Non-event

$n_{cC(M_1)C(M_2)}$

c =1

$C(M_1) \backslash C(M_2)$	1	2	3	4	5	6	7	
1	n_{111}	n_{112}	n_{113}	n_{114}	n_{115}	n_{116}	n_{117}	n_{11+}
2	n_{121}	n_{122}	n_{123}	n_{124}	n_{125}	n_{126}	n_{127}	n_{12+}
3	n_{131}	n_{132}	n_{133}	n_{134}	n_{135}	n_{136}	n_{137}	n_{13+}
4	n_{141}	n_{142}	n_{143}	n_{144}	n_{145}	n_{146}	n_{147}	n_{14+}
5	n_{151}	n_{152}	n_{153}	n_{154}	n_{155}	n_{156}	n_{157}	n_{15+}
6	n_{161}	n_{162}	n_{163}	n_{164}	n_{165}	n_{166}	n_{167}	n_{16+}
7	n_{171}	n_{172}	n_{173}	n_{174}	n_{175}	n_{176}	n_{177}	n_{17+}
	n_{1+1}	n_{1+2}	n_{1+3}	n_{1+4}	n_{1+5}	n_{1+6}	n_{1+7}	n_{1++}

$w_{cC(M_1)C(M_2)}$

	1	2	3	4	5	6	7	
1		-1/6	-2/6	-3/6	-4/6	-5/6	-6/6	
2	1/6		-1/6	-2/6	-3/6	-4/6	-5/6	
3	2/6	1/6		-1/6	-2/6	-3/6	-4/6	
4	3/6	2/6	1/6		-1/6	-2/6	-3/6	
5	4/6	3/6	2/6			-1/6	-2/6	
6	5/6	4/6	3/6	2/6	1/6		-1/6	
7	6/6	5/6	4/6	3/6	2/6	1/6		

c =2

$C(M_1) \backslash C(M_2)$	1	2	3	4	5	6	7	
1	n_{211}	n_{212}	n_{213}	n_{214}	n_{215}	n_{216}	n_{217}	n_{21+}
2	n_{221}	n_{222}	n_{223}	n_{224}	n_{225}	n_{226}	n_{227}	n_{22+}
3	n_{231}	n_{232}	n_{233}	n_{234}	n_{235}	n_{236}	n_{237}	n_{23+}
4	n_{241}	n_{242}	n_{243}	n_{244}	n_{245}	n_{246}	n_{247}	n_{24+}
5	n_{251}	n_{252}	n_{253}	n_{254}	n_{255}	n_{256}	n_{257}	n_{25+}
6	n_{261}	n_{262}	n_{263}	n_{264}	n_{265}	n_{266}	n_{267}	n_{26+}
7	n_{271}	n_{272}	n_{273}	n_{274}	n_{275}	n_{276}	n_{277}	n_{27+}
	n_{2+1}	n_{2+2}	n_{2+3}	n_{2+4}	n_{2+5}	n_{2+6}	n_{2+7}	n_{2++}

	1	2	3	4	5	6	7	
1		1/6		-1/6	-2/6	-3/6	-4/6	
2	-1/6		-1/6	-2/6	-3/6	-4/6	-5/6	
3		1/6		-1/6	-2/6	-3/6	-4/6	
4	1/6	2/6	1/6		-1/6	-2/6	-3/6	
5	2/6	3/6	2/6	1/6		-1/6	-2/6	
6	3/6	4/6	3/6	2/6	1/6		-1/6	
7	4/6	5/6	4/6	3/6	2/6	1/6		

c =3

$C(M_1) \backslash C(M_2)$	1	2	3	4	5	6	7	
1	n_{311}	n_{312}	n_{313}	n_{314}	n_{315}	n_{316}	n_{317}	n_{31+}
2	n_{321}	n_{322}	n_{323}	n_{324}	n_{325}	n_{326}	n_{327}	n_{32+}
3	n_{331}	n_{332}	n_{333}	n_{334}	n_{335}	n_{336}	n_{337}	n_{33+}
4	n_{341}	n_{342}	n_{343}	n_{344}	n_{345}	n_{346}	n_{347}	n_{34+}
5	n_{351}	n_{352}	n_{353}	n_{354}	n_{355}	n_{356}	n_{357}	n_{35+}
6	n_{361}	n_{362}	n_{363}	n_{364}	n_{365}	n_{366}	n_{367}	n_{36+}
7	n_{371}	n_{372}	n_{373}	n_{374}	n_{375}	n_{376}	n_{377}	n_{37+}
	n_{3+1}	n_{3+2}	n_{3+3}	n_{3+4}	n_{3+5}	n_{3+6}	n_{3+7}	n_{3++}

	1	2	3	4	5	6	7	
1		1/6	2/6	1/6		-1/6	-2/6	
2	-1/6		1/6		-1/6	-2/6	-3/6	
3	-2/6	-1/6		-1/6	-2/6	-3/6	-4/6	
4	-1/6		1/6		-1/6	-2/6	-3/6	
5		1/6	2/6	1/6		-1/6	-2/6	
6	1/6	2/6	3/6	2/6	1/6		-1/6	
7	2/6	3/6	4/6	3/6	2/6	1/6		

c =4

$C(M_1)$ \ $C(M_2)$	1	2	3	4	5	6	7	
1	n_{411}	n_{412}	n_{413}	n_{414}	n_{415}	n_{416}	n_{417}	n_{41+}
2	n_{421}	n_{422}	n_{423}	n_{424}	n_{425}	n_{426}	n_{427}	n_{42+}
3	n_{431}	n_{432}	n_{433}	n_{434}	n_{435}	n_{436}	n_{437}	n_{43+}
4	n_{441}	n_{442}	n_{443}	n_{444}	n_{445}	n_{446}	n_{447}	n_{44+}
5	n_{451}	n_{452}	n_{453}	n_{454}	n_{455}	n_{456}	n_{457}	n_{45+}
6	n_{461}	n_{462}	n_{463}	n_{464}	n_{465}	n_{466}	n_{467}	n_{46+}
7	n_{471}	n_{472}	n_{473}	n_{474}	n_{475}	n_{476}	n_{477}	n_{47+}
	n_{4+1}	n_{4+2}	n_{4+3}	n_{4+4}	n_{4+5}	n_{4+6}	n_{4+7}	n_{4++}

	1	2	3	4	5	6	7	
1		1/6	2/6	3/6	2/6	1/6		
2	-1/6		1/6	2/6	1/6		-1/6	
3	-2/6	-1/6		1/6		-1/6	-2/6	
4	-3/6	-2/6	-1/6		-1/6	-2/6	-3/6	
5	-2/6	-1/6		1/6		-1/6	-2/6	
6	-1/6		1/6	2/6	1/6		-1/6	
7		1/6	2/6	3/6	2/6	1/6		

c =5

$C(M_1)$ \ $C(M_2)$	1	2	3	4	5	6	7	
1	n_{511}	n_{512}	n_{513}	n_{514}	n_{515}	n_{516}	n_{517}	n_{51+}
2	n_{521}	n_{522}	n_{523}	n_{524}	n_{525}	n_{526}	n_{527}	n_{52+}
3	n_{531}	n_{532}	n_{533}	n_{534}	n_{535}	n_{536}	n_{537}	n_{53+}
4	n_{541}	n_{542}	n_{543}	n_{544}	n_{545}	n_{546}	n_{547}	n_{54+}
5	n_{551}	n_{552}	n_{553}	n_{554}	n_{555}	n_{556}	n_{557}	n_{55+}
6	n_{561}	n_{562}	n_{563}	n_{564}	n_{565}	n_{566}	n_{567}	n_{56+}
7	n_{571}	n_{572}	n_{573}	n_{574}	n_{575}	n_{576}	n_{577}	n_{57+}
	n_{5+1}	n_{5+2}	n_{5+3}	n_{5+4}	n_{5+5}	n_{5+6}	n_{5+7}	n_{5++}

	1	2	3	4	5	6	7	
1		1/6	2/6	3/6	4/6	3/6	2/6	
2	-1/6		1/6	2/6	3/6	2/6	1/6	
3	-2/6	-1/6		1/6	2/6	1/6		
4	-3/6	-2/6	-1/6		1/6		-1/6	
5	-4/6	-3/6	-2/6	-1/6		-1/6	-2/6	
6	-3/6	-2/6	-1/6		1/6		-1/6	
7	-2/6	-1/6		1/6	2/6	1/6		

c =6

$C(M_1)$ \ $C(M_2)$	1	2	3	4	5	6	7	
1	n_{611}	n_{612}	n_{613}	n_{614}	n_{615}	n_{616}	n_{617}	n_{61+}
2	n_{621}	n_{622}	n_{623}	n_{624}	n_{625}	n_{626}	n_{627}	n_{62+}
3	n_{631}	n_{632}	n_{633}	n_{634}	n_{635}	n_{636}	n_{637}	n_{63+}
4	n_{641}	n_{642}	n_{643}	n_{644}	n_{645}	n_{646}	n_{647}	n_{64+}
5	n_{651}	n_{652}	n_{653}	n_{654}	n_{655}	n_{656}	n_{657}	n_{65+}
6	n_{661}	n_{662}	n_{663}	n_{664}	n_{665}	n_{666}	n_{667}	n_{66+}
7	n_{671}	n_{672}	n_{673}	n_{674}	n_{675}	n_{676}	n_{677}	n_{67+}
	n_{6+1}	n_{6+2}	n_{6+3}	n_{6+4}	n_{6+5}	n_{6+6}	n_{6+7}	n_{6++}

	1	2	3	4	5	6	7	
1		1/6	2/6	3/6	4/6	5/6	4/6	
2	-1/6		1/6	2/6	3/6	4/6	3/6	
3	-2/6	-1/6		1/6	2/6	3/6	2/6	
4	-3/6	-2/6	-1/6		1/6	2/6	1/6	
5	-4/6	-3/6	-2/6	-1/6		1/6		
6	-5/6	-4/6	-3/6	-2/6	-1/6		-1/6	
7	-4/6	-3/6	-2/6	-1/6		1/6		

Eve c =7

nt

$C(M_1)$ \ $C(M_2)$	1	2	3	4	5	6	7	
1	n_{711}	n_{712}	n_{713}	n_{714}	n_{715}	n_{716}	n_{717}	n_{71+}
2	n_{721}	n_{722}	n_{723}	n_{724}	n_{725}	n_{726}	n_{727}	n_{72+}
3	n_{731}	n_{732}	n_{733}	n_{734}	n_{735}	n_{736}	n_{737}	n_{73+}
4	n_{741}	n_{742}	n_{743}	n_{744}	n_{745}	n_{746}	n_{747}	n_{74+}
5	n_{751}	n_{752}	n_{753}	n_{754}	n_{755}	n_{756}	n_{757}	n_{75+}
6	n_{761}	n_{762}	n_{763}	n_{764}	n_{765}	n_{766}	n_{767}	n_{76+}
7	n_{771}	n_{772}	n_{773}	n_{774}	n_{775}	n_{776}	n_{777}	n_{77+}
	n_{7+1}	n_{7+2}	n_{7+3}	n_{7+4}	n_{7+5}	n_{7+6}	n_{7+7}	n_{7++}

	1	2	3	4	5	6	7	
1		1/6	2/6	3/6	4/6	5/6	6/6	
2	-1/6		1/6	2/6	3/6	4/6	56	
3	-2/6	-1/6		1/6	2/6	3/6	4/6	
4	-3/6	-2/6	-1/6		1/6	2/6	3/6	
5	-4/6	-3/6	-2/6	-1/6		1/6	26	
6	-5/6	-4/6	-3/6	-2/6	-1/6		1/6	
7	-6/6	-5/6	-4/6	-3/6	-2/6	-1/6		

Appendix S2: Reclassification table with weight w_2 for ordinal outcomes by M_1 and M_2

1. Two-category outcome

$n_{cC(M_1)C(M_2)}$ $w_{cC(M_1)C(M_2)}$

Non-event

c =1

$C(M_1) \setminus C(M_2)$	1	2	
1	n_{111}	n_{112}	n_{11+}
2	n_{121}	n_{212}	n_{12+}
	n_{1+1}	n_{1+2}	n_{1++}

	1	2	
1		-1	
2	1		

↓

c =2

Event

$C(M_1) \setminus C(M_2)$	1	2	
1	n_{211}	n_{212}	n_{21+}
2	n_{221}	n_{222}	n_{22+}
	n_{2+1}	n_{2+2}	n_{2++}

	1	2	
1		1	
2	-1		

2. Three-category outcome

$n_{cC(M_1)C(M_2)}$ $\quad\quad$ $w_{cC(M_1)C(M_2)}$

Non-event ↓ **Event**

c =1

$C(M_1)\setminus C(M_2)$	1	2	3	
1	n_{111}	n_{112}	n_{113}	n_{11+}
2	n_{121}	n_{122}	n_{123}	n_{12+}
3	n_{131}	n_{132}	n_{133}	n_{13+}
	n_{1+1}	n_{1+2}	n_{1+3}	n_{1++}

	1	2	3	
1		-1	-3/2	
2	1		-1/2	
3	3/2	1/2		

c =2

$C(M_1)\setminus C(M_2)$	1	2	3	
1	n_{211}	n_{212}	n_{213}	n_{21+}
2	n_{221}	n_{222}	n_{223}	n_{22+}
3	n_{231}	n_{232}	n_{233}	n_{23+}
	n_{2+1}	n_{2+2}	n_{2+3}	n_{2++}

	1	2	3	
1		1		
2	-1		-1	
3		1		

c =3

$C(M_1)\setminus C(M_2)$	1	2	3	
1	n_{311}	n_{312}	n_{313}	n_{31+}
2	n_{321}	n_{322}	n_{323}	n_{32+}
3	n_{331}	n_{332}	n_{333}	n_{33+}
	n_{3+1}	n_{3+2}	n_{3+3}	n_{3++}

	1	2	3	
1		1/2	3/2	
2	-1/2		1	
3	-3/2	-1		

3. Four-category outcome

$n_{cC(M_1)C(M_2)}$ $w_{cC(M_1)C(M_2)}$

Non-event

↓

Event

c =1

$C(M_1)$ \ $C(M_2)$	1	2	3	4	
1	n_{111}	n_{112}	n_{113}	n_{114}	n_{11+}
2	n_{121}	n_{122}	n_{123}	n_{124}	n_{12+}
3	n_{131}	n_{132}	n_{133}	n_{134}	n_{13+}
4	n_{141}	n_{142}	n_{143}	n_{144}	n_{14+}
	n_{1+1}	n_{1+2}	n_{1+3}	n_{1+4}	n_{1++}

	1	2	3	4	
1		-1	-4/3	-5/3	
2	1		-1/3	-2/3	
3	4/3	1/3		-1/3	
4	5/3	2/3	1/3		

c =2

$C(M_1)$ \ $C(M_2)$	1	2	3	4	
1	n_{211}	n_{212}	n_{213}	n_{214}	n_{21+}
2	n_{221}	n_{222}	n_{223}	n_{224}	n_{22+}
3	n_{231}	n_{232}	n_{233}	n_{234}	n_{23+}
4	n_{241}	n_{242}	n_{243}	n_{244}	n_{24+}
	n_{2+1}	n_{2+2}	n_{2+3}	n_{2+4}	n_{2++}

	1	2	3	4	
1		1		-1/3	
2	-1		-1	-4/3	
3		1		-1/3	
4	1/3	4/3	1/3		

c =3

$C(M_1)$ \ $C(M_2)$	1	2	3	4	
1	n_{311}	n_{312}	n_{313}	n_{314}	n_{31+}
2	n_{321}	n_{322}	n_{323}	n_{324}	n_{32+}
3	n_{331}	n_{332}	n_{333}	n_{334}	n_{33+}
4	n_{341}	n_{342}	n_{343}	n_{344}	n_{34+}
	n_{3+1}	n_{3+2}	n_{3+3}	n_{3+4}	n_{3++}

	1	2	3	4	
1		1/3	4/3	1/3	
2	-1/3		1		
3	-4/3	-1		-1	
4	-1/3		1		

c =4

$C(M_1)$ \ $C(M_2)$	1	2	3	4	
1	n_{411}	n_{412}	n_{413}	n_{414}	n_{41+}
2	n_{421}	n_{422}	n_{423}	n_{424}	n_{42+}
3	n_{431}	n_{432}	n_{433}	n_{434}	n_{43+}
4	n_{441}	n_{442}	n_{443}	n_{444}	n_{44+}
	n_{4+1}	n_{4+2}	n_{4+3}	n_{4+4}	n_{4++}

	1	2	3	4	
1		1/3	2/3	5/3	
2	-1/3		1/3	4/3	
3	-2/3	-1/3		1	
4	-5/3	-4/3	-1		

4. Five-category outcome

Non-event

$n_{cC(M_1)C(M_2)}$

c =1

$C(M_1) \setminus C(M_2)$	1	2	3	4	5	
1	n_{111}	n_{112}	n_{113}	n_{114}	n_{115}	n_{11+}
2	n_{121}	n_{122}	n_{123}	n_{124}	n_{125}	n_{12+}
3	n_{131}	n_{132}	n_{133}	n_{134}	n_{135}	n_{13+}
4	n_{141}	n_{142}	n_{143}	n_{144}	n_{145}	n_{14+}
5	n_{151}	n_{152}	n_{153}	n_{154}	n_{155}	n_{15+}
	n_{1+1}	n_{1+2}	n_{1+3}	n_{1+4}	n_{1+5}	n_{1++}

$w_{cC(M_1)C(M_2)}$

	1	2	3	4	5	
1		-1	-5/4	-6/4	-7/4	
2	1		-1/4	-2/4	-3/4	
3	5/4	1/4		-1/4	-2/4	
4	6/4	2/4	1/4		-1/4	
5	7/4	3/4	2/4	1/4		

c =2

$C(M_1) \setminus C(M_2)$	1	2	3	4	5	
1	n_{211}	n_{212}	n_{213}	n_{214}	n_{215}	n_{21+}
2	n_{221}	n_{222}	n_{223}	n_{224}	n_{225}	n_{22+}
3	n_{231}	n_{232}	n_{233}	n_{234}	n_{235}	n_{23+}
4	n_{241}	n_{242}	n_{243}	n_{244}	n_{245}	n_{24+}
5	n_{251}	n_{252}	n_{253}	n_{254}	n_{255}	n_{25+}
	n_{2+1}	n_{2+2}	n_{2+3}	n_{2+4}	n_{2+5}	n_{2++}

	1	2	3	4	5	
1		1		-1/4	-2/4	
2	-1		-1	-5/4	-6/4	
3		1		-1/4	-2/4	
4	1/4	5/4	1/4		-1/4	
5	2/4	6/4	2/4	1/4		

c =3

$C(M_1) \setminus C(M_2)$	1	2	3	4	5	
1	n_{311}	n_{312}	n_{313}	n_{314}	n_{315}	n_{31+}
2	n_{321}	n_{322}	n_{323}	n_{324}	n_{325}	n_{32+}
3	n_{331}	n_{332}	n_{333}	n_{334}	n_{335}	n_{33+}
4	n_{341}	n_{342}	n_{343}	n_{344}	n_{345}	n_{34+}
5	n_{351}	n_{352}	n_{353}	n_{354}	n_{355}	n_{35+}
	n_{3+1}	n_{3+2}	n_{3+3}	n_{3+4}	n_{3+5}	n_{3++}

	1	2	3	4	5	
1		1/4	5/4	1/4		
2	-1/4		1		-1/4	
3	-5/4	-1		-1	-5/4	
4	-1/4		1		-1/4	
5		1/4	5/4	1/4		

c =4

$C(M_1) \setminus C(M_2)$	1	2	3	4	5	
1	n_{411}	n_{412}	n_{413}	n_{414}	n_{415}	n_{41+}
2	n_{421}	n_{422}	n_{423}	n_{424}	n_{425}	n_{42+}
3	n_{431}	n_{432}	n_{433}	n_{434}	n_{435}	n_{43+}
4	n_{441}	n_{442}	n_{443}	n_{444}	n_{445}	n_{44+}
5	n_{451}	n_{452}	n_{453}	n_{454}	n_{455}	n_{45+}
	n_{4+1}	n_{4+2}	n_{4+3}	n_{4+4}	n_{4+5}	n_{4++}

	1	2	3	4	5	
1		1/4	2/4	6/4	2/4	
2	-1/4		1/4	5/4	1/4	
3	-2/4	-1/4		1		
4	-6/4	-5/4	-1		-1	
5	-2/4	-1/4		1		

c =5

Event	$C(M_1) \setminus C(M_2)$	1	2	3	4	5	
	1	n_{511}	n_{512}	n_{513}	n_{514}	n_{515}	n_{51+}
	2	n_{521}	n_{522}	n_{523}	n_{524}	n_{525}	n_{52+}
	3	n_{531}	n_{532}	n_{533}	n_{534}	n_{535}	n_{53+}
	4	n_{541}	n_{542}	n_{543}	n_{544}	n_{545}	n_{54+}
	5	n_{551}	n_{552}	n_{553}	n_{554}	n_{555}	n_{55+}
		n_{5+1}	n_{5+2}	n_{5+3}	n_{5+4}	n_{5+5}	n_{5++}

	1	2	3	4	5	
1		1/4	2/4	3/4	7/4	
2	-1/4		1/4	2/4	6/4	
3	-2/4	-1/4		1/4	5/4	
4	-3/4	-2/4	-1/4		1	
5	-7/4	-6/4	-5/4	-1		

5. Six-category outcome

$n_{cC(M_1)C(M_2)}$ $w_{cC(M_1)C(M_2)}$

Non-event

c =1

$C(M_1)$ \ $C(M_2)$	1	2	3	4	5	6	
1	n_{111}	n_{112}	n_{113}	n_{114}	n_{115}	n_{116}	n_{11+}
2	n_{121}	n_{122}	n_{123}	n_{124}	n_{125}	n_{126}	n_{12+}
3	n_{131}	n_{132}	n_{133}	n_{134}	n_{135}	n_{136}	n_{13+}
4	n_{141}	n_{142}	n_{143}	n_{144}	n_{145}	n_{146}	n_{14+}
5	n_{151}	n_{152}	n_{153}	n_{154}	n_{155}	n_{156}	n_{15+}
6	n_{161}	n_{162}	n_{163}	n_{164}	n_{165}	n_{166}	n_{16+}
	n_{1+1}	n_{1+2}	n_{1+3}	n_{1+4}	n_{1+5}	n_{1+6}	n_{1++}

	1	2	3	4	5	6
1		-1	-6/5	-7/5	-8/5	-9/5
2	1		-1/5	-2/5	-3/5	-4/5
3	6/5	1/5		-1/5	-2/5	-3/5
4	7/5	2/5	1/5		-1/5	-2/5
5	8/5	3/5	2/5	1/5		-1/5
6	9/5	4/5	3/5	2/5	1/5	

c =2

$C(M_1)$ \ $C(M_2)$	1	2	3	4	5	6	
1	n_{211}	n_{212}	n_{213}	n_{214}	n_{215}	n_{216}	n_{21+}
2	n_{221}	n_{222}	n_{223}	n_{224}	n_{225}	n_{226}	n_{22+}
3	n_{231}	n_{232}	n_{233}	n_{234}	n_{235}	n_{236}	n_{23+}
4	n_{241}	n_{242}	n_{243}	n_{244}	n_{245}	n_{246}	n_{24+}
5	n_{251}	n_{252}	n_{253}	n_{254}	n_{255}	n_{256}	n_{25+}
6	n_{261}	n_{262}	n_{263}	n_{264}	n_{265}	n_{266}	n_{26+}
	n_{2+1}	n_{2+2}	n_{2+3}	n_{2+4}	n_{2+5}	n_{2+6}	n_{2++}

	1	2	3	4	5	6
1		1		-1/5	-2/5	-3/5
2	-1		-1	-6/5	-7/5	-8/5
3		1		-1/5	-2/5	-3/5
4	1/5	6/5	1/5		-1/5	-2/5
5	2/5	7/5	2/5	1/5		-1/5
6	3/5	8/5	3/5	2/5	1/5	

c =3

$C(M_1)$ \ $C(M_2)$	1	2	3	4	5	6	
1	n_{311}	n_{312}	n_{313}	n_{314}	n_{315}	n_{316}	n_{31+}
2	n_{321}	n_{322}	n_{323}	n_{324}	n_{325}	n_{326}	n_{32+}
3	n_{331}	n_{332}	n_{333}	n_{334}	n_{335}	n_{336}	n_{33+}
4	n_{341}	n_{342}	n_{343}	n_{344}	n_{345}	n_{346}	n_{34+}
5	n_{351}	n_{352}	n_{353}	n_{354}	n_{355}	n_{356}	n_{35+}
6	n_{361}	n_{362}	n_{363}	n_{364}	n_{365}	n_{366}	n_{36+}
	n_{3+1}	n_{3+2}	n_{3+3}	n_{3+4}	n_{3+5}	n_{3+6}	n_{3++}

	1	2	3	4	5	6
1		1/5	6/5	1/5		-1/5
2	-1/5		1		-1/5	-2/5
3	-6/5	-1		-1	-6/5	-7/5
4	-1/5		1		-1/5	-2/5
5		1/5	6/5	1/5		-1/5
6	1/5	2/5	7/5	2/5	1/5	

c =4

$C(M_1)$ \ $C(M_2)$	1	2	3	4	5	6	
1	n_{411}	n_{412}	n_{413}	n_{414}	n_{415}	n_{416}	n_{41+}
2	n_{421}	n_{422}	n_{423}	n_{424}	n_{425}	n_{426}	n_{42+}
3	n_{431}	n_{432}	n_{433}	n_{434}	n_{435}	n_{436}	n_{43+}
4	n_{441}	n_{442}	n_{443}	n_{444}	n_{445}	n_{446}	n_{44+}
5	n_{451}	n_{452}	n_{453}	n_{454}	n_{455}	n_{456}	n_{45+}
6	n_{461}	n_{462}	n_{463}	n_{464}	n_{465}	n_{466}	n_{46+}
	n_{4+1}	n_{4+2}	n_{4+3}	n_{4+4}	n_{4+5}	n_{4+6}	n_{4++}

	1	2	3	4	5	6	
1		1/5	2/5	7/5	2/5	1/5	
2	-1/5		1/5	6/5	1/5		
3	-2/5	-1/5		1		-1/5	
4	-7/5	-6/5	-1		-1	-6/5	
5	-2/5	-1/5		1		-1/5	
6	-1/5		1/5	6/5	1/5		

c =5

$C(M_1)$ \ $C(M_2)$	1	2	3	4	5	6	
1	n_{511}	n_{512}	n_{513}	n_{514}	n_{515}	n_{516}	n_{51+}
2	n_{521}	n_{522}	n_{523}	n_{524}	n_{525}	n_{526}	n_{52+}
3	n_{531}	n_{532}	n_{533}	n_{534}	n_{535}	n_{536}	n_{53+}
4	n_{541}	n_{542}	n_{543}	n_{544}	n_{545}	n_{546}	n_{54+}
5	n_{551}	n_{552}	n_{553}	n_{554}	n_{555}	n_{556}	n_{55+}
6	n_{561}	n_{562}	n_{563}	n_{564}	n_{565}	n_{566}	n_{56+}
	n_{5+1}	n_{5+2}	n_{5+3}	n_{5+4}	n_{5+5}	n_{5+5}	n_{5++}

	1	2	3	4	5	6	
1		1/5	2/5	3/5	8/5	3/5	
2	-1/5		1/5	2/5	7/5	2/5	
3	-2/5	-1/5		1/5	6/5	1/5	
4	-3/5	-2/5	-1/5		1		
5	-8/5	-7/5	-6/5	-1		-1	
6	-3/5	-2/5	-1/5		1		

c =6

$C(M_1)$ \ $C(M_2)$	1	2	3	4	5	6	
1	n_{611}	n_{612}	n_{613}	n_{614}	n_{615}	n_{616}	n_{61+}
2	n_{621}	n_{622}	n_{623}	n_{624}	n_{625}	n_{626}	n_{62+}
3	n_{631}	n_{632}	n_{633}	n_{634}	n_{635}	n_{636}	n_{63+}
4	n_{641}	n_{642}	n_{643}	n_{644}	n_{645}	n_{645}	n_{64+}
5	n_{651}	n_{652}	n_{653}	n_{654}	n_{655}	n_{656}	n_{65+}
6	n_{661}	n_{662}	n_{663}	n_{664}	n_{665}	n_{666}	n_{66+}
	n_{6+1}	n_{6+2}	n_{6+3}	n_{6+4}	n_{6+5}	n_{6+5}	n_{6++}

	1	2	3	4	5	6	
1		1/5	2/5	3/5	4/5	9/5	
2	-1/5		1/5	2/5	3/5	8/5	
3	-2/5	-1/5		1/5	2/5	7/5	
4	-3/5	-2/5	-1/5		1/5	6/5	
5	-4/5	-3/5	-2/5	-1/5		1	
6	-9/5	-8/5	-7/5	-6/5	-1		

6. Seven-category outcome

Non-event

$n_{cC(M_1)C(M_2)}$

c =1

$C(M_1) \setminus C(M_2)$	1	2	3	4	5	6	7	
1	n_{111}	n_{112}	n_{113}	n_{114}	n_{115}	n_{116}	n_{117}	n_{11+}
2	n_{121}	n_{122}	n_{123}	n_{124}	n_{125}	n_{126}	n_{127}	n_{12+}
3	n_{131}	n_{132}	n_{133}	n_{134}	n_{135}	n_{136}	n_{137}	n_{13+}
4	n_{141}	n_{142}	n_{143}	n_{144}	n_{145}	n_{146}	n_{147}	n_{14+}
5	n_{151}	n_{152}	n_{153}	n_{154}	n_{155}	n_{156}	n_{157}	n_{15+}
6	n_{161}	n_{162}	n_{163}	n_{164}	n_{165}	n_{166}	n_{167}	n_{16+}
7	n_{171}	n_{172}	n_{173}	n_{174}	n_{175}	n_{176}	n_{177}	n_{17+}
	n_{1+1}	n_{1+2}	n_{1+3}	n_{1+4}	n_{1+5}	n_{1+6}	n_{1+7}	n_{1++}

$w_{cC(M_1)C(M_2)}$

	1	2	3	4	5	6	7	
1		-1	-7/6	-8/6	-9/6	-10/6	-11/6	
2	1		-1/6	-2/6	-3/6	-4/6	-5/6	
3	7/6	1/6		-1/6	-2/6	-3/6	-4/6	
4	8/6	2/6	1/6		-1/6	-2/6	-3/6	
5	9/6	3/6	2/6	1/6		-1/6	-2/6	
6	10/6	4/6	3/6	2/6	1/6		-1/6	
7	11/6	5/6	4/6	3/6	2/6	1/6		

c =2

$C(M_1) \setminus C(M_2)$	1	2	3	4	5	6	7	
1	n_{211}	n_{212}	n_{213}	n_{214}	n_{215}	n_{216}	n_{217}	n_{21+}
2	n_{221}	n_{222}	n_{223}	n_{224}	n_{225}	n_{226}	n_{227}	n_{22+}
3	n_{231}	n_{232}	n_{233}	n_{234}	n_{235}	n_{236}	n_{237}	n_{23+}
4	n_{241}	n_{242}	n_{243}	n_{244}	n_{245}	n_{246}	n_{247}	n_{24+}
5	n_{251}	n_{252}	n_{253}	n_{254}	n_{255}	n_{256}	n_{257}	n_{25+}
6	n_{261}	n_{262}	n_{263}	n_{264}	n_{265}	n_{266}	n_{267}	n_{26+}
7	n_{271}	n_{272}	n_{273}	n_{274}	n_{275}	n_{276}	n_{277}	n_{27+}
	n_{2+1}	n_{2+2}	n_{2+3}	n_{2+4}	n_{2+5}	n_{2+6}	n_{2+7}	n_{2++}

	1	2	3	4	5	6	7	
1		1		-1/6	-2/6	-3/6	-4/6	
2	-1		-1	-7/6	-8/6	-9/6	-10/6	
3		1		-1/6	-2/6	-3/6	-4/6	
4	1/6	7/6	1/6		-1/6	-2/6	-3/6	
5	2/6	8/6	2/6	1/6		-1/6	-2/6	
6	3/6	9/6	3/6	2/6	1/6		-1/6	
7	4/6	10/6	4/6	3/6	2/6	1/6		

c =3

$C(M_1) \setminus C(M_2)$	1	2	3	4	5	6	7	
1	n_{311}	n_{312}	n_{313}	n_{314}	n_{315}	n_{316}	n_{317}	n_{31+}
2	n_{321}	n_{322}	n_{323}	n_{324}	n_{325}	n_{326}	n_{327}	n_{32+}
3	n_{331}	n_{332}	n_{333}	n_{334}	n_{335}	n_{336}	n_{337}	n_{33+}
4	n_{341}	n_{342}	n_{343}	n_{344}	n_{345}	n_{346}	n_{347}	n_{34+}
5	n_{351}	n_{352}	n_{353}	n_{354}	n_{355}	n_{356}	n_{357}	n_{35+}
6	n_{361}	n_{362}	n_{363}	n_{364}	n_{365}	n_{366}	n_{367}	n_{36+}
7	n_{371}	n_{372}	n_{373}	n_{374}	n_{375}	n_{376}	n_{377}	n_{37+}
	n_{3+1}	n_{3+2}	n_{3+3}	n_{3+4}	n_{3+5}	n_{3+6}	n_{3+7}	n_{3++}

	1	2	3	4	5	6	7	
1		1/6	7/6	1/6		-1/6	-2/6	
2	-1/6		1		-1/6	-2/6	-3/6	
3	-7/6	-1		-1	-7/6	-8/6	-9/6	
4	-1/6		1		-1/6	-2/6	-3/6	
5		1/6	7/6	1/6		-1/6	-2/6	
6	1/6	2/6	8/6	2/6	1/6		-1/6	
7	2/6	3/6	9/6	3/6	2/6	1/6		

c =4

$C(M_1)$ \ $C(M_2)$	1	2	3	4	5	6	7	
1	n_{411}	n_{412}	n_{413}	n_{414}	n_{415}	n_{416}	n_{417}	n_{41+}
2	n_{421}	n_{422}	n_{423}	n_{424}	n_{425}	n_{426}	n_{427}	n_{42+}
3	n_{431}	n_{432}	n_{433}	n_{434}	n_{435}	n_{436}	n_{437}	n_{43+}
4	n_{441}	n_{442}	n_{443}	n_{444}	n_{445}	n_{446}	n_{447}	n_{44+}
5	n_{451}	n_{452}	n_{453}	n_{454}	n_{455}	n_{456}	n_{457}	n_{45+}
6	n_{461}	n_{462}	n_{463}	n_{464}	n_{465}	n_{466}	n_{467}	n_{46+}
7	n_{471}	n_{472}	n_{473}	n_{474}	n_{475}	n_{476}	n_{477}	n_{47+}
	n_{4+1}	n_{4+2}	n_{4+3}	n_{4+4}	n_{4+5}	n_{4+6}	n_{4+7}	n_{4++}

	1	2	3	4	5	6	7	
1		1/6	2/6	8/6	2/6	1/6		
2	-1/6		1/6	7/6	1/6		-1/6	
3	-2/6	-1/6		1		-1/6	-2/6	
4	-8/6	-7/6	-1		-1	-7/6	-8/6	
5	-2/6	-1/6		1		-1/6	-2/6	
6	-1/6		1/6	7/6	1/6		-1/6	
7		1/6	2/6	8/6	2/6	1/6		

c =5

$C(M_1)$ \ $C(M_2)$	1	2	3	4	5	6	7	
1	n_{511}	n_{512}	n_{513}	n_{514}	n_{515}	n_{516}	n_{517}	n_{51+}
2	n_{521}	n_{522}	n_{523}	n_{524}	n_{525}	n_{526}	n_{527}	n_{52+}
3	n_{531}	n_{532}	n_{533}	n_{534}	n_{535}	n_{536}	n_{537}	n_{53+}
4	n_{541}	n_{542}	n_{543}	n_{544}	n_{545}	n_{546}	n_{547}	n_{54+}
5	n_{551}	n_{552}	n_{553}	n_{554}	n_{555}	n_{556}	n_{557}	n_{55+}
6	n_{561}	n_{562}	n_{563}	n_{564}	n_{565}	n_{566}	n_{567}	n_{56+}
7	n_{571}	n_{572}	n_{573}	n_{574}	n_{575}	n_{576}	n_{577}	n_{57+}
	n_{5+1}	n_{5+2}	n_{5+3}	n_{5+4}	n_{5+5}	n_{5+6}	n_{5+7}	n_{5++}

	1	2	3	4	5	6	7	
1		1/6	2/6	3/6	9/6	3/6	2/6	
2	-1/6		1/6	2/6	8/6	2/6	1/6	
3	-2/6	-1/6		1/6	7/6	1/6		
4	-3/6	-2/6	-1/6		1		-1/6	
5	-9/6	-8/6	-7/6	-1		-1	-7/6	
6	-3/6	-2/6	-1/6		1		-1/6	
7	-2/6	-1/6		1/6	7/6	1/6		

c =6

$C(M_1)$ \ $C(M_2)$	1	2	3	4	5	6	7	
1	n_{611}	n_{612}	n_{613}	n_{614}	n_{615}	n_{616}	n_{617}	n_{61+}
2	n_{621}	n_{622}	n_{623}	n_{624}	n_{625}	n_{626}	n_{627}	n_{62+}
3	n_{631}	n_{632}	n_{633}	n_{634}	n_{635}	n_{636}	n_{637}	n_{63+}
4	n_{641}	n_{642}	n_{643}	n_{644}	n_{645}	n_{646}	n_{647}	n_{64+}
5	n_{651}	n_{652}	n_{653}	n_{654}	n_{655}	n_{656}	n_{657}	n_{65+}
6	n_{661}	n_{662}	n_{663}	n_{664}	n_{665}	n_{666}	n_{667}	n_{66+}
7	n_{671}	n_{672}	n_{673}	n_{674}	n_{675}	n_{676}	n_{677}	n_{67+}
	n_{6+1}	n_{6+2}	n_{6+3}	n_{6+4}	n_{6+5}	n_{6+6}	n_{6+7}	n_{6++}

	1	2	3	4	5	6	7	
1		1/6	2/6	3/6	4/6	10/6	4/6	
2	-1/6		1/6	2/6	3/6	9/6	3/6	
3	-2/6	-1/6		1/6	2/6	8/6	2/6	
4	-3/6	-2/6	-1/6		1/6	7/6	1/6	
5	-4/6	-3/6	-2/6	-1/6		1		
6	-10/6	-9/6	-8/6	-7/6	-1		-1	
7	-4/6	-3/6	-2/6	-1/6		1		

c =7

	$C(M_1) \setminus C(M_2)$	1	2	3	4	5	6	7	
	1	n_{711}	n_{712}	n_{713}	n_{714}	n_{715}	n_{716}	n_{717}	n_{71+}
Eve	2	n_{721}	n_{722}	n_{723}	n_{724}	n_{725}	n_{726}	n_{727}	n_{72+}
nt	3	n_{731}	n_{732}	n_{733}	n_{734}	n_{735}	n_{736}	n_{737}	n_{73+}
	4	n_{741}	n_{742}	n_{743}	n_{744}	n_{745}	n_{746}	n_{747}	n_{74+}
	5	n_{751}	n_{752}	n_{753}	n_{754}	n_{755}	n_{756}	n_{757}	n_{75+}
	6	n_{761}	n_{762}	n_{763}	n_{764}	n_{765}	n_{766}	n_{767}	n_{76+}
	7	n_{771}	n_{772}	n_{773}	n_{774}	n_{775}	n_{776}	n_{777}	n_{77+}
		n_{7+1}	n_{7+2}	n_{7+3}	n_{7+4}	n_{7+5}	n_{7+6}	n_{7+7}	n_{7++}

	1	2	3	4	5	6	7	
1		1/6	2/6	3/6	4/6	5/6	11/6	
2	-1/6		1/6	2/6	3/6	4/6	10/6	
3	-2/6	-1/6		1/6	2/6	3/6	9/6	
4	-3/6	-2/6	-1/6		1/6	2/6	8/6	
5	-4/6	-3/6	-2/6	-1/6		1/6	7/6	
6	-5/6	-4/6	-3/6	-2/6	-1/6		1	
7	- 11/6	- 10/6	-9/6	-8/6	-7/6	-1		

Appendix S3: Reclassification table with weight w_3 for ordinal outcomes by M_1 and M_2

1. Two-category outcome

$n_{cC(M_1)C(M_2)}$

Non-event

$c=1$

$C(M_1) \setminus C(M_2)$	1	2	
1	n_{111}	n_{112}	n_{11+}
2	n_{121}	n_{212}	n_{12+}
	n_{1+1}	n_{1+2}	n_{1++}

$w_{cC(M_1)C(M_2)}$

	1	2	
1		-1	
2	1		

↓

Event

$c=2$

$C(M_1) \setminus C(M_2)$	1	2	
1	n_{211}	n_{212}	n_{21+}
2	n_{221}	n_{222}	n_{22+}
	n_{2+1}	n_{2+2}	n_{2++}

	1	2	
1		1	
2	-1		

2. Three-category outcome

$n_{cC(M_1)C(M_2)}$ | $w_{cC(M_1)C(M_2)}$

Non-event

c =1

$C(M_1) \setminus C(M_2)$	1	2	3	
1	n_{111}	n_{112}	n_{113}	n_{11+}
2	n_{121}	n_{122}	n_{123}	n_{12+}
3	n_{131}	n_{132}	n_{133}	n_{13+}
	n_{1+1}	n_{1+2}	n_{1+3}	n_{1++}

	1	2	3	
1		-1	-1	
2	1		-1/2	
3	1	1/2		

↓

c =2

$C(M_1) \setminus C(M_2)$	1	2	3	
1	n_{211}	n_{212}	n_{213}	n_{21+}
2	n_{221}	n_{222}	n_{223}	n_{22+}
3	n_{231}	n_{232}	n_{233}	n_{23+}
	n_{2+1}	n_{2+2}	n_{2+3}	n_{2++}

	1	2	3	
1		1		
2	-1		-1	
3		1		

Event

c =3

$C(M_1) \setminus C(M_2)$	1	2	3	
1	n_{311}	n_{312}	n_{313}	n_{31+}
2	n_{321}	n_{322}	n_{323}	n_{32+}
3	n_{331}	n_{332}	n_{333}	n_{33+}
	n_{3+1}	n_{3+2}	n_{3+3}	n_{3++}

	1	2	3	
1		1/2	1	
2	-1/2		1	
3	-1	-1		

3. Four-category outcome

$n_{cC(M_1)C(M_2)}$ $\qquad\qquad$ $w_{cC(M_1)C(M_2)}$

Non-event

f=1

$C(M_1) \setminus C(M_2)$	1	2	3	4	
1	n_{111}	n_{112}	n_{113}	n_{114}	n_{11+}
2	n_{121}	n_{122}	n_{123}	n_{124}	n_{12+}
3	n_{131}	n_{132}	n_{133}	n_{134}	n_{13+}
4	n_{141}	n_{142}	n_{143}	n_{144}	n_{14+}
	n_{1+1}	n_{1+2}	n_{1+3}	n_{1+4}	n_{1++}

	1	2	3	4	
1		-1	-1	-1	
2	1		-1/3	-2/3	
3	1	1/3		-1/3	
4	1	2/3	1/3		

c =2

$C(M_1) \setminus C(M_2)$	1	2	3	4	
1	n_{211}	n_{212}	n_{213}	n_{214}	n_{21+}
2	n_{221}	n_{222}	n_{223}	n_{224}	n_{22+}
3	n_{231}	n_{232}	n_{233}	n_{234}	n_{23+}
4	n_{241}	n_{242}	n_{243}	n_{244}	n_{24+}
	n_{2+1}	n_{2+2}	n_{2+3}	n_{2+4}	n_{2++}

	1	2	3	4	
1		1		-1/3	
2	-1		-1	-1	
3		1		-1/3	
4	1/3	1	1/3		

↓

c =3

$C(M_1) \setminus C(M_2)$	1	2	3	4	
1	n_{311}	n_{312}	n_{313}	n_{314}	n_{31+}
2	n_{321}	n_{322}	n_{323}	n_{324}	n_{32+}
3	n_{331}	n_{332}	n_{333}	n_{334}	n_{33+}
4	n_{341}	n_{342}	n_{343}	n_{344}	n_{34+}
	n_{3+1}	n_{3+2}	n_{3+3}	n_{3+4}	n_{3++}

	1	2	3	4	
1		1/3	1	1/3	
2	-1/3		1		
3	-1	-1		-1	
4	-1/3		1		

Event

c =4

$C(M_1) \setminus C(M_2)$	1	2	3	4	
1	n_{411}	n_{412}	n_{413}	n_{414}	n_{41+}
2	n_{421}	n_{422}	n_{423}	n_{424}	n_{42+}
3	n_{431}	n_{432}	n_{433}	n_{434}	n_{43+}
4	n_{441}	n_{442}	n_{443}	n_{444}	n_{44+}
	n_{4+1}	n_{4+2}	n_{4+3}	n_{4+4}	n_{4++}

	1	2	3	4	
1		1/3	2/3	1	
2	-1/3		1/3	1	
3	-2/3	-1/3		1	
4	-1	-1	-1		

4. Five-category outcome

$n_{cC(M_1)C(M_2)}$ $w_{cC(M_1)C(M_2)}$

Non-event

↓

c =1

$C(M_1) \setminus C(M_2)$	1	2	3	4	5	
1	n_{111}	n_{112}	n_{113}	n_{114}	n_{115}	n_{11+}
2	n_{121}	n_{122}	n_{123}	n_{124}	n_{125}	n_{12+}
3	n_{131}	n_{132}	n_{133}	n_{134}	n_{135}	n_{13+}
4	n_{141}	n_{142}	n_{143}	n_{144}	n_{145}	n_{14+}
5	n_{151}	n_{152}	n_{153}	n_{154}	n_{155}	n_{15+}
	n_{1+1}	n_{1+2}	n_{1+3}	n_{1+4}	n_{1+5}	n_{1++}

	1	2	3	4	5
1		-1	-1	-1	-1
2	1		-1/4	-2/4	-3/4
3	1	1/4		-1/4	-2/4
4	1	2/4	1/4		-1/4
5	1	3/4	2/4	1/4	

c =2

$C(M_1) \setminus C(M_2)$	1	2	3	4	5	
1	n_{211}	n_{212}	n_{213}	n_{214}	n_{215}	n_{21+}
2	n_{221}	n_{222}	n_{223}	n_{224}	n_{225}	n_{22+}
3	n_{231}	n_{232}	n_{233}	n_{234}	n_{235}	n_{23+}
4	n_{241}	n_{242}	n_{243}	n_{244}	n_{245}	n_{24+}
5	n_{251}	n_{252}	n_{253}	n_{254}	n_{255}	n_{25+}
	n_{2+1}	n_{2+2}	n_{2+3}	n_{2+4}	n_{2+5}	n_{2++}

	1	2	3	4	5
1		1		-1/4	-2/4
2	-1		-1	-1	-1
3		1		-1/4	-2/4
4	1/4	1	1/4		-1/4
5	2/4	1	2/4	1/4	

c =3

$C(M_1) \setminus C(M_2)$	1	2	3	4	5	
1	n_{311}	n_{312}	n_{313}	n_{314}	n_{315}	n_{31+}
2	n_{321}	n_{322}	n_{323}	n_{324}	n_{325}	n_{32+}
3	n_{331}	n_{332}	n_{333}	n_{334}	n_{335}	n_{33+}
4	n_{341}	n_{342}	n_{343}	n_{344}	n_{345}	n_{34+}
5	n_{351}	n_{352}	n_{353}	n_{354}	n_{355}	n_{35+}
	n_{3+1}	n_{3+2}	n_{3+3}	n_{3+4}	n_{3+5}	n_{3++}

	1	2	3	4	5
1		1/4	1	1/4	
2	-1/4		1		-1/4
3	-1	-1		-1	-1
4	-1/4		1		-1/4
5		1/4	1	1/4	

c =4

$C(M_1) \setminus C(M_2)$	1	2	3	4	5	
1	n_{411}	n_{412}	n_{413}	n_{414}	n_{415}	n_{41+}
2	n_{421}	n_{422}	n_{423}	n_{424}	n_{425}	n_{42+}
3	n_{431}	n_{432}	n_{433}	n_{434}	n_{435}	n_{43+}
4	n_{441}	n_{442}	n_{443}	n_{444}	n_{445}	n_{44+}
5	n_{451}	n_{452}	n_{453}	n_{454}	n_{455}	n_{45+}
	n_{4+1}	n_{4+2}	n_{4+3}	n_{4+4}	n_{4+5}	n_{4++}

	1	2	3	4	5
1		1/4	2/4	1	2/4
2	-1/4		1/4	1	1/4
3	-2/4	-1/4		1	
4	-1	-1	-1		-1
5	-2/4	-1/4		1	

c =5

Event	$C(M_1) \setminus C(M_2)$	1	2	3	4	5	
	1	n_{511}	n_{512}	n_{513}	n_{514}	n_{515}	n_{51+}
	2	n_{521}	n_{522}	n_{523}	n_{524}	n_{525}	n_{52+}
	3	n_{531}	n_{532}	n_{533}	n_{534}	n_{535}	n_{53+}
	4	n_{541}	n_{542}	n_{543}	n_{544}	n_{545}	n_{54+}
	5	n_{551}	n_{552}	n_{553}	n_{554}	n_{555}	n_{55+}
		n_{5+1}	n_{5+2}	n_{5+3}	n_{5+4}	n_{5+5}	n_{5++}

	1	2	3	4	5	
1		1/4	2/4	3/4	1	
2	-1/4		1/4	2/4	1	
3	-2/4	-1/4		1/4	1	
4	-3/4	-2/4	-1/4		1	
5	-1	-1	-1	-1		

5. Six-category outcome

$n_{cC(M_1)C(M_2)}$ $\quad$ $w_{cC(M_1)C(M_2)}$

Non-event

c =1

$C(M_1)$ \ $C(M_2)$	1	2	3	4	5	6	
1	n_{111}	n_{112}	n_{113}	n_{114}	n_{115}	n_{116}	n_{11+}
2	n_{121}	n_{122}	n_{123}	n_{124}	n_{125}	n_{126}	n_{12+}
3	n_{131}	n_{132}	n_{133}	n_{134}	n_{135}	n_{136}	n_{13+}
4	n_{141}	n_{142}	n_{143}	n_{144}	n_{145}	n_{146}	n_{14+}
5	n_{151}	n_{152}	n_{153}	n_{154}	n_{155}	n_{156}	n_{15+}
6	n_{161}	n_{162}	n_{163}	n_{164}	n_{165}	n_{166}	n_{16+}
	n_{1+1}	n_{1+2}	n_{1+3}	n_{1+4}	n_{1+5}	n_{1+6}	n_{1++}

	1	2	3	4	5	6
1		-1	-1	-1	-1	-1
2	1		-1/5	-2/5	-3/5	-4/5
3	1	1/5		-1/5	-2/5	-3/5
4	1	2/5	1/5		-1/5	-2/5
5	1	3/5	2/5	1/5		-1/5
6	1	4/5	3/5	2/5	1/5	

c =2

$C(M_1)$ \ $C(M_2)$	1	2	3	4	5	6	
1	n_{211}	n_{212}	n_{213}	n_{214}	n_{215}	n_{216}	n_{21+}
2	n_{221}	n_{222}	n_{223}	n_{224}	n_{225}	n_{226}	n_{22+}
3	n_{231}	n_{232}	n_{233}	n_{234}	n_{235}	n_{236}	n_{23+}
4	n_{241}	n_{242}	n_{243}	n_{244}	n_{245}	n_{246}	n_{24+}
5	n_{251}	n_{252}	n_{253}	n_{254}	n_{255}	n_{256}	n_{25+}
6	n_{261}	n_{262}	n_{263}	n_{264}	n_{265}	n_{266}	n_{26+}
	n_{2+1}	n_{2+2}	n_{2+3}	n_{2+4}	n_{2+5}	n_{2+6}	n_{2++}

	1	2	3	4	5	6
1		1		-1/5	-2/5	-3/5
2	-1		-1	-1	-1	-1
3		1		-1/5	-2/5	-3/5
4	1/5	1	1/5		-1/5	-2/5
5	2/5	1	2/5	1/5		-1/5
6	3/5	1	3/5	2/5	1/5	

c =3

$C(M_1)$ \ $C(M_2)$	1	2	3	4	5	6	
1	n_{311}	n_{312}	n_{313}	n_{314}	n_{315}	n_{316}	n_{31+}
2	n_{321}	n_{322}	n_{323}	n_{324}	n_{325}	n_{326}	n_{32+}
3	n_{331}	n_{332}	n_{333}	n_{334}	n_{335}	n_{336}	n_{33+}
4	n_{341}	n_{342}	n_{343}	n_{344}	n_{345}	n_{346}	n_{34+}
5	n_{351}	n_{352}	n_{353}	n_{354}	n_{355}	n_{356}	n_{35+}
6	n_{361}	n_{362}	n_{363}	n_{364}	n_{365}	n_{366}	n_{36+}
	n_{3+1}	n_{3+2}	n_{3+3}	n_{3+4}	n_{3+5}	n_{3+6}	n_{3++}

	1	2	3	4	5	6
1		1/5	1	1/5		-1/5
2	-1/5		1		-1/5	-2/5
3	-1	-1		-1	-1	-1
4	-1/5		1		-1/5	-2/5
5		1/5	1	1/5		-1/5
6	1/5	2/5	1	2/5	1/5	

c =4

$C(M_1)$ \ $C(M_2)$	1	2	3	4	5	6	
1	n_{411}	n_{412}	n_{413}	n_{414}	n_{415}	n_{416}	n_{41+}
2	n_{421}	n_{422}	n_{423}	n_{424}	n_{425}	n_{426}	n_{42+}
3	n_{431}	n_{432}	n_{433}	n_{434}	n_{435}	n_{436}	n_{43+}
4	n_{441}	n_{442}	n_{443}	n_{444}	n_{445}	n_{446}	n_{44+}
5	n_{451}	n_{452}	n_{453}	n_{454}	n_{455}	n_{456}	n_{45+}
6	n_{461}	n_{462}	n_{463}	n_{464}	n_{465}	n_{466}	n_{46+}
	n_{4+1}	n_{4+2}	n_{4+3}	n_{4+4}	n_{4+5}	n_{4+6}	n_{4++}

	1	2	3	4	5	6	
1		1/5	2/5	1	2/5	1/5	
2	-1/5		1/5	1	1/5		
3	-2/5	-1/5		1		-1/5	
4	-1	-1	-1		-1	-1	
5	-2/5	-1/5		1		-1/5	
6	-1/5		1/5	1	1/5		

c =5

$C(M_1)$ \ $C(M_2)$	1	2	3	4	5	6	
1	n_{511}	n_{512}	n_{513}	n_{514}	n_{515}	n_{516}	n_{51+}
2	n_{521}	n_{522}	n_{523}	n_{524}	n_{525}	n_{526}	n_{52+}
3	n_{531}	n_{532}	n_{533}	n_{534}	n_{535}	n_{536}	n_{53+}
4	n_{541}	n_{542}	n_{543}	n_{544}	n_{545}	n_{546}	n_{54+}
5	n_{551}	n_{552}	n_{553}	n_{554}	n_{555}	n_{556}	n_{55+}
6	n_{561}	n_{562}	n_{563}	n_{564}	n_{565}	n_{566}	n_{56+}
	n_{5+1}	n_{5+2}	n_{5+3}	n_{5+4}	n_{5+5}	n_{5+5}	n_{5++}

	1	2	3	4	5	6	
1		1/5	2/5	3/5	1	3/5	
2	-1/5		1/5	2/5	1	2/5	
3	-2/5	-1/5		1/5	1	1/5	
4	-3/5	-2/5	-1/5		1		
5	-1	-1	-1	-1		-1	
6	-3/5	-2/5	-1/5		1		

c =6

$C(M_1)$ \ $C(M_2)$	1	2	3	4	5	6	
1	n_{611}	n_{612}	n_{613}	n_{614}	n_{615}	n_{616}	n_{61+}
2	n_{621}	n_{622}	n_{623}	n_{624}	n_{625}	n_{626}	n_{62+}
3	n_{631}	n_{632}	n_{633}	n_{634}	n_{635}	n_{636}	n_{63+}
4	n_{641}	n_{642}	n_{643}	n_{644}	n_{645}	n_{645}	n_{64+}
5	n_{651}	n_{652}	n_{653}	n_{654}	n_{655}	n_{656}	n_{65+}
6	n_{661}	n_{662}	n_{663}	n_{664}	n_{665}	n_{666}	n_{66+}
	n_{6+1}	n_{6+2}	n_{6+3}	n_{6+4}	n_{6+5}	n_{6+5}	n_{6++}

	1	2	3	4	5	6	
1		1/5	2/5	3/5	4/5	1	
2	-1/5		1/5	2/5	3/5	1	
3	-2/5	-1/5		1/5	2/5	1	
4	-3/5	-2/5	-1/5		1/5	1	
5	-4/5	-3/5	-2/5	-1/5		1	
6	-1	-1	-1	-1	-1		

5. Seven-category outcome

$n_{cC(M_1)C(M_2)}$

$w_{cC(M_1)C(M_2)}$

Non-event

c =1

$C(M_1)$ \ $C(M_2)$	1	2	3	4	5	6	7	
1	n_{111}	n_{112}	n_{113}	n_{114}	n_{115}	n_{116}	n_{117}	n_{11+}
2	n_{121}	n_{122}	n_{123}	n_{124}	n_{125}	n_{126}	n_{127}	n_{12+}
3	n_{131}	n_{132}	n_{133}	n_{134}	n_{135}	n_{136}	n_{137}	n_{13+}
4	n_{141}	n_{142}	n_{143}	n_{144}	n_{145}	n_{146}	n_{147}	n_{14+}
5	n_{151}	n_{152}	n_{153}	n_{154}	n_{155}	n_{156}	n_{157}	n_{15+}
6	n_{161}	n_{162}	n_{163}	n_{164}	n_{165}	n_{166}	n_{167}	n_{16+}
7	n_{171}	n_{172}	n_{173}	n_{174}	n_{175}	n_{176}	n_{177}	n_{17+}
	n_{1+1}	n_{1+2}	n_{1+3}	n_{1+4}	n_{1+5}	n_{1+6}	n_{1+7}	n_{1++}

	1	2	3	4	5	6	7	
1		-1	-1	-1	-1	-1	-1	
2	1		-1/6	-2/6	-3/6	-4/6	-5/6	
3	1	1/6		-1/6	-2/6	-3/6	-4/6	
4	1	2/6	1/6		-1/6	-2/6	-3/6	
5	1	3/6	2/6	1/6		-1/6	-2/6	
6	1	4/6	3/6	2/6	1/6		-1/6	
7	1	5/6	4/6	3/6	2/6	1/6		

c =2

$C(M_1)$ \ $C(M_2)$	1	2	3	4	5	6	7	
1	n_{211}	n_{212}	n_{213}	n_{214}	n_{215}	n_{216}	n_{217}	n_{21+}
2	n_{221}	n_{222}	n_{223}	n_{224}	n_{225}	n_{226}	n_{227}	n_{22+}
3	n_{231}	n_{232}	n_{233}	n_{234}	n_{235}	n_{236}	n_{237}	n_{23+}
4	n_{241}	n_{242}	n_{243}	n_{244}	n_{245}	n_{246}	n_{247}	n_{24+}
5	n_{251}	n_{252}	n_{253}	n_{254}	n_{255}	n_{256}	n_{257}	n_{25+}
6	n_{261}	n_{262}	n_{263}	n_{264}	n_{265}	n_{266}	n_{267}	n_{26+}
7	n_{271}	n_{272}	n_{273}	n_{274}	n_{275}	n_{276}	n_{277}	n_{27+}
	n_{2+1}	n_{2+2}	n_{2+3}	n_{2+4}	n_{2+5}	n_{2+6}	n_{2+7}	n_{2++}

	1	2	3	4	5	6	7	
1		1		-1/6	-2/6	-3/6	-4/6	
2	-1		-1	-1	-1	-1	-1	
3		1		-1/6	-2/6	-3/6	-4/6	
4	1/6	1	1/6		-1/6	-2/6	-3/6	
5	2/6	1	2/6	1/6		-1/6	-2/6	
6	3/6	1	3/6	2/6	1/6		-1/6	
7	4/6	1	4/6	3/6	2/6	1/6		

c =3

$C(M_1)$ \ $C(M_2)$	1	2	3	4	5	6	7	
1	n_{311}	n_{312}	n_{313}	n_{314}	n_{315}	n_{316}	n_{317}	n_{31+}
2	n_{321}	n_{322}	n_{323}	n_{324}	n_{325}	n_{326}	n_{327}	n_{32+}
3	n_{331}	n_{332}	n_{333}	n_{334}	n_{335}	n_{336}	n_{337}	n_{33+}
4	n_{341}	n_{342}	n_{343}	n_{344}	n_{345}	n_{346}	n_{347}	n_{34+}
5	n_{351}	n_{352}	n_{353}	n_{354}	n_{355}	n_{356}	n_{357}	n_{35+}
6	n_{361}	n_{362}	n_{363}	n_{364}	n_{365}	n_{366}	n_{367}	n_{36+}
7	n_{371}	n_{372}	n_{373}	n_{374}	n_{375}	n_{376}	n_{377}	n_{37+}
	n_{3+1}	n_{3+2}	n_{3+3}	n_{3+4}	n_{3+5}	n_{3+6}	n_{3+7}	n_{3++}

	1	2	3	4	5	6	7	
1		1/6	1	1/6		-1/6	-2/6	
2	-1/6		1		-1/6	-2/6	-3/6	
3	-1	-1		-1	-1	-1	-1	
4	-1/6		1		-1/6	-2/6	-3/6	
5		1/6	1	1/6		-1/6	-2/6	
6	1/6	2/6	1	2/6	1/6		-1/6	
7	2/6	3/6	1	3/6	2/6	1/6		

c =4

$C(M_1)$ \ $C(M_2)$	1	2	3	4	5	6	7	
1	n_{411}	n_{412}	n_{413}	n_{414}	n_{415}	n_{416}	n_{417}	n_{41+}
2	n_{421}	n_{422}	n_{423}	n_{424}	n_{425}	n_{426}	n_{427}	n_{42+}
3	n_{431}	n_{432}	n_{433}	n_{434}	n_{435}	n_{436}	n_{437}	n_{43+}
4	n_{441}	n_{442}	n_{443}	n_{444}	n_{445}	n_{446}	n_{447}	n_{44+}
5	n_{451}	n_{452}	n_{453}	n_{454}	n_{455}	n_{456}	n_{457}	n_{45+}
6	n_{461}	n_{462}	n_{463}	n_{464}	n_{465}	n_{466}	n_{467}	n_{46+}
7	n_{471}	n_{472}	n_{473}	n_{474}	n_{475}	n_{476}	n_{477}	n_{47+}
	n_{4+1}	n_{4+2}	n_{4+3}	n_{4+4}	n_{4+5}	n_{4+6}	n_{4+7}	n_{4++}

	1	2	3	4	5	6	7	
1		1/6	2/6	1	2/6	1/6		
2	-1/6		1/6	1	1/6		-1/6	
3	-2/6	-1/6		1		-1/6	-2/6	
4	-1	-1	-1		-1	-1	-1	
5	-2/6	-1/6		1		-1/6	-2/6	
6	-1/6		1/6	1	1/6		-1/6	
7		1/6	2/6	1	2/6	1/6		

c =5

$C(M_1)$ \ $C(M_2)$	1	2	3	4	5	6	7	
1	n_{511}	n_{512}	n_{513}	n_{514}	n_{515}	n_{516}	n_{517}	n_{51+}
2	n_{521}	n_{522}	n_{523}	n_{524}	n_{525}	n_{526}	n_{527}	n_{52+}
3	n_{531}	n_{532}	n_{533}	n_{534}	n_{535}	n_{536}	n_{537}	n_{53+}
4	n_{541}	n_{542}	n_{543}	n_{544}	n_{545}	n_{546}	n_{547}	n_{54+}
5	n_{551}	n_{552}	n_{553}	n_{554}	n_{555}	n_{556}	n_{557}	n_{55+}
6	n_{561}	n_{562}	n_{563}	n_{564}	n_{565}	n_{566}	n_{567}	n_{56+}
7	n_{571}	n_{572}	n_{573}	n_{574}	n_{575}	n_{576}	n_{577}	n_{57+}
	n_{5+1}	n_{5+2}	n_{5+3}	n_{5+4}	n_{5+5}	n_{5+6}	n_{5+7}	n_{5++}

	1	2	3	4	5	6	7	
1		1/6	2/6	3/6	1	3/6	2/6	
2	-1/6		1/6	2/6	1	2/6	1/6	
3	-2/6	-1/6		1/6	1	1/6		
4	-3/6	-2/6	-1/6		1		-1/6	
5	-1	-1	-1	-1		-1	-1	
6	-3/6	-2/6	-1/6		1		-1/6	
7	-2/6	-1/6		1/6	1	1/6		

c =6

$C(M_1)$ \ $C(M_2)$	1	2	3	4	5	6	7	
1	n_{611}	n_{612}	n_{613}	n_{614}	n_{615}	n_{616}	n_{617}	n_{61+}
2	n_{621}	n_{622}	n_{623}	n_{624}	n_{625}	n_{626}	n_{627}	n_{62+}
3	n_{631}	n_{632}	n_{633}	n_{634}	n_{635}	n_{636}	n_{637}	n_{63+}
4	n_{641}	n_{642}	n_{643}	n_{644}	n_{645}	n_{646}	n_{647}	n_{64+}
5	n_{651}	n_{652}	n_{653}	n_{654}	n_{655}	n_{656}	n_{657}	n_{65+}
6	n_{661}	n_{662}	n_{663}	n_{664}	n_{665}	n_{666}	n_{667}	n_{66+}
7	n_{671}	n_{672}	n_{673}	n_{674}	n_{675}	n_{676}	n_{677}	n_{67+}
	n_{6+1}	n_{6+2}	n_{6+3}	n_{6+4}	n_{6+5}	n_{6+6}	n_{6+7}	n_{6++}

	1	2	3	4	5	6	7	
1		1/6	2/6	3/6	4/6	1	4/6	
2	-1/6		1/6	2/6	3/6	1	3/6	
3	-2/6	-1/6		1/6	2/6	1	2/6	
4	-3/6	-2/6	-1/6		1/6	1	1/6	
5	-4/6	-3/6	-2/6	-1/6		1		
6	-1	-1	-1	-1	-1		-1	
7	-4/6	-3/6	-2/6	-1/6		1		

Eve c =7

nt

$C(M_1)$ \ $C(M_2)$	1	2	3	4	5	6	7	
1	n_{711}	n_{712}	n_{713}	n_{714}	n_{715}	n_{716}	n_{717}	n_{71+}
2	n_{721}	n_{722}	n_{723}	n_{724}	n_{725}	n_{726}	n_{727}	n_{72+}
3	n_{731}	n_{732}	n_{733}	n_{734}	n_{735}	n_{736}	n_{737}	n_{73+}
4	n_{741}	n_{742}	n_{743}	n_{744}	n_{745}	n_{746}	n_{747}	n_{74+}
5	n_{751}	n_{752}	n_{753}	n_{754}	n_{755}	n_{756}	n_{757}	n_{75+}
6	n_{761}	n_{762}	n_{763}	n_{764}	n_{765}	n_{766}	n_{767}	n_{76+}
7	n_{771}	n_{772}	n_{773}	n_{774}	n_{775}	n_{776}	n_{777}	n_{77+}
	n_{7+1}	n_{7+2}	n_{7+3}	n_{7+4}	n_{7+5}	n_{7+6}	n_{7+7}	n_{7++}

	1	2	3	4	5	6	7	
1		1/6	2/6	3/6	4/6	5/6	1	
2	-1/6		1/6	2/6	3/6	4/6	1	
3	-2/6	-1/6		1/6	2/6	3/6	1	
4	-3/6	-2/6	-1/6		1/6	2/6	1	
5	-4/6	-3/6	-2/6	-1/6		1/6	1	
6	-5/6	-4/6	-3/6	-2/6	-1/6		1	
7	-1	-1	-1	-1	-1	-1		

Appendix S4: $\boldsymbol{a}_c$ and $\boldsymbol{b}_c$ with weight w_1 and w_2 for ordinal outcomes by M_1 and M_2

	c	$\boldsymbol{a}_c$ (w_1)	$\boldsymbol{a}_c$ (w_2)	$\boldsymbol{b}_c$
2group	1	$[1]$	$[1]$	$[n_{1+1} - n_{11+}]$
	2	$[1]$	$[1]$	$[n_{2+2} - n_{22+}]$
3group	1	$\begin{bmatrix} 2/2 \\ 1/2 \end{bmatrix}$	$\begin{bmatrix} 3/2 \\ 1/2 \end{bmatrix}$	$\begin{bmatrix} n_{1+1} - n_{11+} \\ n_{1+2} - n_{12+} \end{bmatrix}$
	2	$[1/2]$	$[1]$	$[n_{2+2} - n_{22+}]$
	3	$\begin{bmatrix} 2/2 \\ 1/2 \end{bmatrix}$	$\begin{bmatrix} 3/2 \\ 1/2 \end{bmatrix}$	$\begin{bmatrix} n_{3+3} - n_{33+} \\ n_{3+2} - n_{32+} \end{bmatrix}$
4group	1	$\begin{bmatrix} 3/3 \\ 2/3 \\ 1/3 \end{bmatrix}$	$\begin{bmatrix} 5/3 \\ 2/3 \\ 1/3 \end{bmatrix}$	$\begin{bmatrix} n_{1+1} - n_{11+} \\ n_{1+2} - n_{12+} \\ n_{1+3} - n_{13+} \end{bmatrix}$
	2	$\begin{bmatrix} 1/3 \\ 2/3 \\ 3/3 \end{bmatrix}$	$\begin{bmatrix} 1/3 \\ 4/3 \\ 1/3 \end{bmatrix}$	$\begin{bmatrix} n_{2+1} - n_{21+} \\ n_{2+2} - n_{22+} \\ n_{2+3} - n_{23+} \end{bmatrix}$
	3	$\begin{bmatrix} 1/3 \\ 2/3 \\ 3/3 \end{bmatrix}$	$\begin{bmatrix} 1/3 \\ 4/3 \\ 1/3 \end{bmatrix}$	$\begin{bmatrix} n_{3+4} - n_{34+} \\ n_{3+3} - n_{33+} \\ n_{3+2} - n_{32+} \end{bmatrix}$
	4	$\begin{bmatrix} 3/3 \\ 2/3 \\ 1/3 \end{bmatrix}$	$\begin{bmatrix} 5/3 \\ 2/3 \\ 1/3 \end{bmatrix}$	$\begin{bmatrix} n_{4+4} - n_{44+} \\ n_{4+3} - n_{43+} \\ n_{4+2} - n_{42+} \end{bmatrix}$
5group	1	$\begin{bmatrix} 4/4 \\ 3/4 \\ 2/4 \\ 1/4 \end{bmatrix}$	$\begin{bmatrix} 7/4 \\ 3/4 \\ 2/4 \\ 1/4 \end{bmatrix}$	$\begin{bmatrix} n_{1+1} - n_{11+} \\ n_{1+2} - n_{12+} \\ n_{1+3} - n_{13+} \\ n_{1+4} - n_{14+} \end{bmatrix}$
	2	$\begin{bmatrix} 2/4 \\ 3/4 \\ 2/4 \\ 1/4 \end{bmatrix}$	$\begin{bmatrix} 2/4 \\ 6/4 \\ 2/4 \\ 1/4 \end{bmatrix}$	$\begin{bmatrix} n_{2+1} - n_{21+} \\ n_{2+2} - n_{22+} \\ n_{2+3} - n_{23+} \\ n_{2+4} - n_{24+} \end{bmatrix}$
	3	$\begin{bmatrix} 1/4 \\ 2/4 \\ 1/4 \end{bmatrix}$	$\begin{bmatrix} 1/4 \\ 5/4 \\ 1/4 \end{bmatrix}$	$\begin{bmatrix} n_{3+2} - n_{32+} \\ n_{3+3} - n_{33+} \\ n_{3+4} - n_{34+} \end{bmatrix}$
	4	$\begin{bmatrix} 2/4 \\ 3/4 \\ 2/4 \\ 1/4 \end{bmatrix}$	$\begin{bmatrix} 2/4 \\ 6/4 \\ 2/4 \\ 1/4 \end{bmatrix}$	$\begin{bmatrix} n_{4+5} - n_{45+} \\ n_{4+4} - n_{44+} \\ n_{4+3} - n_{43+} \\ n_{4+2} - n_{42+} \end{bmatrix}$

	5	$\begin{bmatrix}4/4\\3/4\\2/4\\1/4\end{bmatrix}$	$\begin{bmatrix}7/4\\3/4\\2/4\\1/4\end{bmatrix}$	$\begin{bmatrix}n_{5+5}-n_{55+}\\n_{5+4}-n_{54+}\\n_{5+3}-n_{53+}\\n_{5+2}-n_{52+}\end{bmatrix}$
6group	1	$\begin{bmatrix}5/5\\4/5\\3/5\\2/5\\1/5\end{bmatrix}$	$\begin{bmatrix}9/5\\4/5\\3/5\\2/5\\1/5\end{bmatrix}$	$\begin{bmatrix}n_{1+1}-n_{11+}\\n_{1+2}-n_{12+}\\n_{1+3}-n_{13+}\\n_{1+4}-n_{14+}\\n_{1+5}-n_{15+}\end{bmatrix}$
	2	$\begin{bmatrix}3/5\\4/5\\3/5\\2/5\\1/5\end{bmatrix}$	$\begin{bmatrix}3/5\\8/5\\3/5\\2/5\\1/5\end{bmatrix}$	$\begin{bmatrix}n_{2+1}-n_{21+}\\n_{2+2}-n_{22+}\\n_{2+3}-n_{23+}\\n_{2+4}-n_{24+}\\n_{2+5}-n_{25+}\end{bmatrix}$
	3	$\begin{bmatrix}1/5\\2/5\\3/5\\2/5\\1/5\end{bmatrix}$	$\begin{bmatrix}1/5\\2/5\\7/5\\2/5\\1/5\end{bmatrix}$	$\begin{bmatrix}n_{3+1}-n_{31+}\\n_{3+2}-n_{32+}\\n_{3+3}-n_{33+}\\n_{3+4}-n_{34+}\\n_{3+5}-n_{35+}\end{bmatrix}$
	4	$\begin{bmatrix}1/5\\2/5\\3/5\\2/5\\1/5\end{bmatrix}$	$\begin{bmatrix}1/5\\2/5\\7/5\\2/5\\1/5\end{bmatrix}$	$\begin{bmatrix}n_{4+6}-n_{46+}\\n_{4+5}-n_{45+}\\n_{4+4}-n_{44+}\\n_{4+3}-n_{43+}\\n_{4+2}-n_{42+}\end{bmatrix}$
	5	$\begin{bmatrix}3/5\\4/5\\3/5\\2/5\\1/5\end{bmatrix}$	$\begin{bmatrix}3/5\\8/5\\3/5\\2/5\\1/5\end{bmatrix}$	$\begin{bmatrix}n_{5+6}-n_{56+}\\n_{5+5}-n_{55+}\\n_{5+4}-n_{54+}\\n_{5+3}-n_{53+}\\n_{5+2}-n_{52+}\end{bmatrix}$
	6	$\begin{bmatrix}5/5\\4/5\\3/5\\2/5\\1/5\end{bmatrix}$	$\begin{bmatrix}9/5\\4/5\\3/5\\2/5\\1/5\end{bmatrix}$	$\begin{bmatrix}n_{6+6}-n_{66+}\\n_{6+5}-n_{65+}\\n_{6+4}-n_{64+}\\n_{6+3}-n_{63+}\\n_{6+2}-n_{62+}\end{bmatrix}$
7group	1	$\begin{bmatrix}6/6\\5/6\\4/6\\3/6\\2/6\\1/6\end{bmatrix}$	$\begin{bmatrix}11/6\\5/6\\4/6\\3/6\\2/6\\1/6\end{bmatrix}$	$\begin{bmatrix}n_{1+1}-n_{11+}\\n_{1+2}-n_{12+}\\n_{1+3}-n_{13+}\\n_{1+4}-n_{14+}\\n_{1+5}-n_{15+}\\n_{1+6}-n_{16+}\end{bmatrix}$
	2	$\begin{bmatrix}4/6\\5/6\\4/6\\3/6\\2/6\\1/6\end{bmatrix}$	$\begin{bmatrix}4/6\\10/6\\4/6\\3/6\\2/6\\1/6\end{bmatrix}$	$\begin{bmatrix}n_{2+1}-n_{21+}\\n_{2+2}-n_{22+}\\n_{2+3}-n_{23+}\\n_{2+4}-n_{24+}\\n_{2+5}-n_{25+}\\n_{2+6}-n_{26+}\end{bmatrix}$

3	$\begin{bmatrix} 2/6 \\ 3/6 \\ 4/6 \\ 3/6 \\ 2/6 \\ 1/6 \end{bmatrix}$	$\begin{bmatrix} 2/6 \\ 3/6 \\ 9/6 \\ 3/6 \\ 2/6 \\ 1/6 \end{bmatrix}$	$\begin{bmatrix} n_{3+1} - n_{31+} \\ n_{3+2} - n_{32+} \\ n_{3+3} - n_{33+} \\ n_{3+4} - n_{34+} \\ n_{3+5} - n_{35+} \\ n_{3+6} - n_{36+} \end{bmatrix}$
4	$\begin{bmatrix} 1/6 \\ 2/6 \\ 3/6 \\ 2/6 \\ 1/6 \end{bmatrix}$	$\begin{bmatrix} 1/6 \\ 2/6 \\ 8/6 \\ 2/6 \\ 1/6 \end{bmatrix}$	$\begin{bmatrix} n_{4+2} - n_{42+} \\ n_{4+3} - n_{43+} \\ n_{4+4} - n_{44+} \\ n_{4+5} - n_{45+} \\ n_{4+6} - n_{46+} \end{bmatrix}$
5	$\begin{bmatrix} 2/6 \\ 3/6 \\ 4/6 \\ 3/6 \\ 2/6 \\ 1/6 \end{bmatrix}$	$\begin{bmatrix} 2/6 \\ 3/6 \\ 9/6 \\ 3/6 \\ 2/6 \\ 1/6 \end{bmatrix}$	$\begin{bmatrix} n_{5+7} - n_{57+} \\ n_{5+6} - n_{56+} \\ n_{5+5} - n_{55+} \\ n_{5+4} - n_{54+} \\ n_{5+3} - n_{53+} \\ n_{5+2} - n_{52+} \end{bmatrix}$
6	$\begin{bmatrix} 4/6 \\ 5/6 \\ 4/6 \\ 3/6 \\ 2/6 \\ 1/6 \end{bmatrix}$	$\begin{bmatrix} 4/6 \\ 10/6 \\ 4/6 \\ 3/6 \\ 2/6 \\ 1/6 \end{bmatrix}$	$\begin{bmatrix} n_{6+7} - n_{67+} \\ n_{6+6} - n_{66+} \\ n_{6+5} - n_{65+} \\ n_{6+4} - n_{64+} \\ n_{6+3} - n_{63+} \\ n_{6+2} - n_{62+} \end{bmatrix}$
7	$\begin{bmatrix} 6/6 \\ 5/6 \\ 4/6 \\ 3/6 \\ 2/6 \\ 1/6 \end{bmatrix}$	$\begin{bmatrix} 11/6 \\ 5/6 \\ 4/6 \\ 3/6 \\ 2/6 \\ 1/6 \end{bmatrix}$	$\begin{bmatrix} n_{7+7} - n_{77+} \\ n_{7+6} - n_{76+} \\ n_{7+5} - n_{75+} \\ n_{7+4} - n_{74+} \\ n_{7+3} - n_{73+} \\ n_{7+2} - n_{72+} \end{bmatrix}$

국 문 요 약

순위형 결과변수를 위한 NRI

임상에서 기존에 알려진 인자 외에 새로운 인자를 찾는 것은 진단의 정확도 및 의사들의 의사결정 측면에서 굉장히 중요하다. NRI 는 새로운 인자의 추가된 예측력을 평가하고 제시하는 척도이다. 이분형 결과 변수(예: 질병상태), 생존 결과 변수(예: 전체 생존율과 무병생존율), 다범주 결과 변수(예: 질병 종류)와 같은 다양한 결과 변수에 대해 NRI 는 진단의 정확도의 향상을 평가하는데 사용한다.

최근에 진단의 평가, 삶의 질 척도, 통증이나 증상의 중증도, 병의 진행 단계 등과 같은 순위형 자료로 많은 의사결정이 이루어지고 있다. 순위형 결과변수가 중요한 결과물 중 하나임에도 불구하고, 순서를 고려한 NRI 는 존재하지 않는다.

본 연구에서는 이분형 결과변수와 다범주 결과변수의 NRI 이론을 기반으로 3 가지 방법으로 weight 를 고려한 순위형 결과변수에 대한 NRI 를 제안하고, Stuart-Maxwell test 와 Bhapkar’s test 의 분산 추정 이론을 기반으로 SE 를 제안하였다.

본 연구에서는 제안하는 방법과 기존에 존재하는 방법을 평가하기 위해 시뮬레이션을 수행하였다. 순위형 자료에 대한 VUS 와 다범주 자료에 대한 RI,

NRI, 임의로 나눈 이분형 자료에 대한 AUC 를 비교하였다. 시뮬레이션 결과 제안한 방법이 다른 방법들에 비해 높은 coverage rate 을 보였고, 특히 Delong method 보다 더 높은 coverage rate 을 보였다. 또한 상대 위험도의 측면에서 다범주 모형 보다는 순위형 모형에서 더 안정적인 결과를 제시하였다. Nakas 의 방법은 복잡하고, 계산 시간이 오래 리는 반면, 제안한 방법은 단순하고, 계산 시간이 짧게 소요되었다. 제안한 방법 중에는 Stuart-Maxwell test 를 이용하여 분산을 추정한 방법이 Bhapkar test 를 이용하여 분산을 추정한 방법보다 높은 coverage rate 을 보였다.

제안한 방법의 유용성을 알아보기 위해, 녹내장 자료와 대뇌 죽상경화증 자료에 적용하였다. 녹내장 자료의 경우 결과변수를 임의로 두 군으로 나누어, Pencina 와 Delong 의 방법을 이용할 경우, NRI, ΔAUC 에 의해 새로운 인자의 예측력의 향상을 입증할 수 없었으나, 기존의 순위형 결과 변수인 세 군을 유지하였을 때, 제안한 방법으로는 새로운 인자의 예측력의 향상을 입증할 수 있었다. 대뇌 죽상경화증 자료의 경우 결과 변수를 2~7 개까지 다양하게 고려해 보았다. 이 경우에는 기존의 결과 변수 범주인 7 개를 유지하였을 때, 새로 추가된 인자의 유용성을 입증할 수 있었다.

분산의 독립성을 가정하지 않고, 분산을 추정하는 방법에 대한 연구 및 IDI 에 대한 연구가 추가적으로 이루어져야 하고, 조금 더 다양한 유병율을 가정하여 시뮬레이션을 수행하여 결과를 더 일반화 할 필요가 있다.

핵심 되는 말: 새로운 인자, 순위형 종속변수, NRI, VUS